PHYSICS- INFORMED MACHINE LEARNING FOR COSMOLOGICAL SIMULATIONS

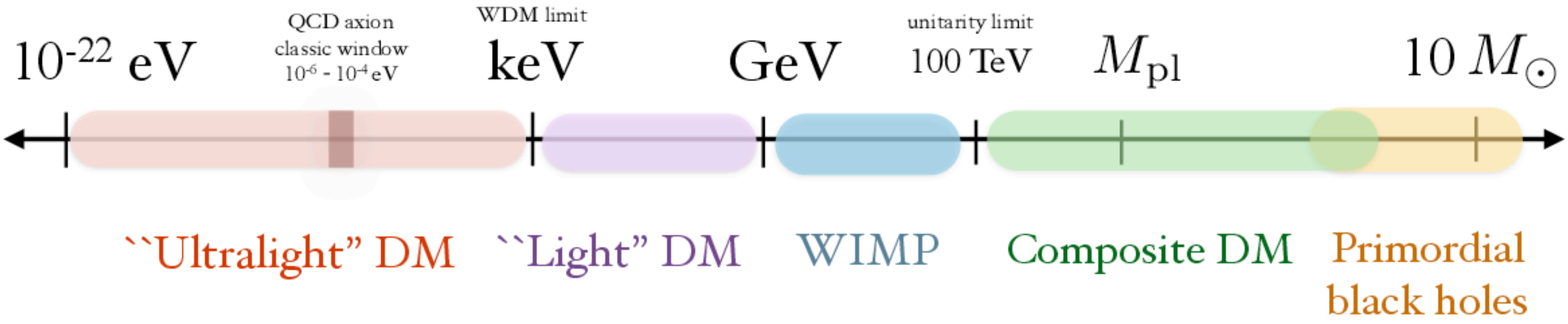
Emma Tolley

AI+ Astronomy
Hangzhou, China
22 October 2025

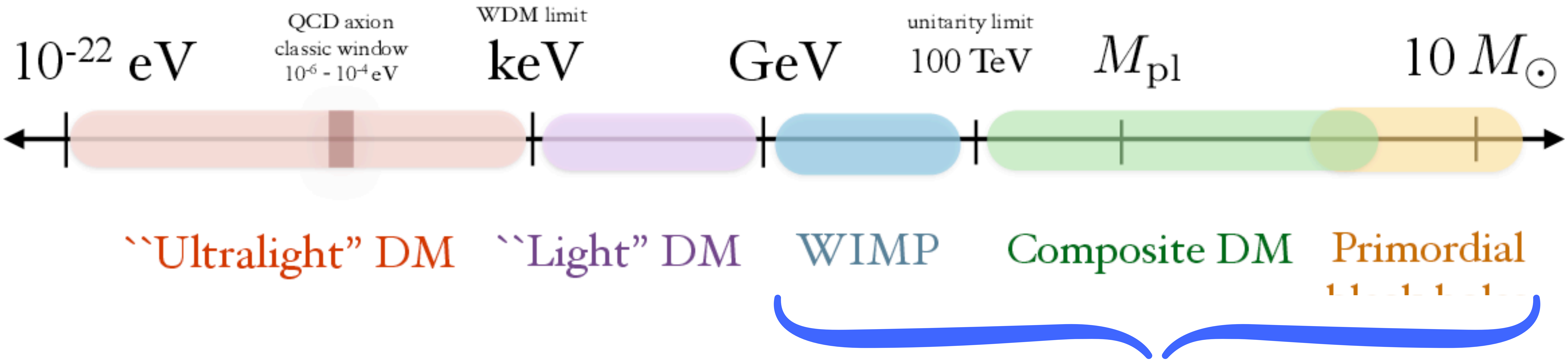
EPFL
Swiss National Science Foundation

MATTER CONTENT OF THE UNIVERSE

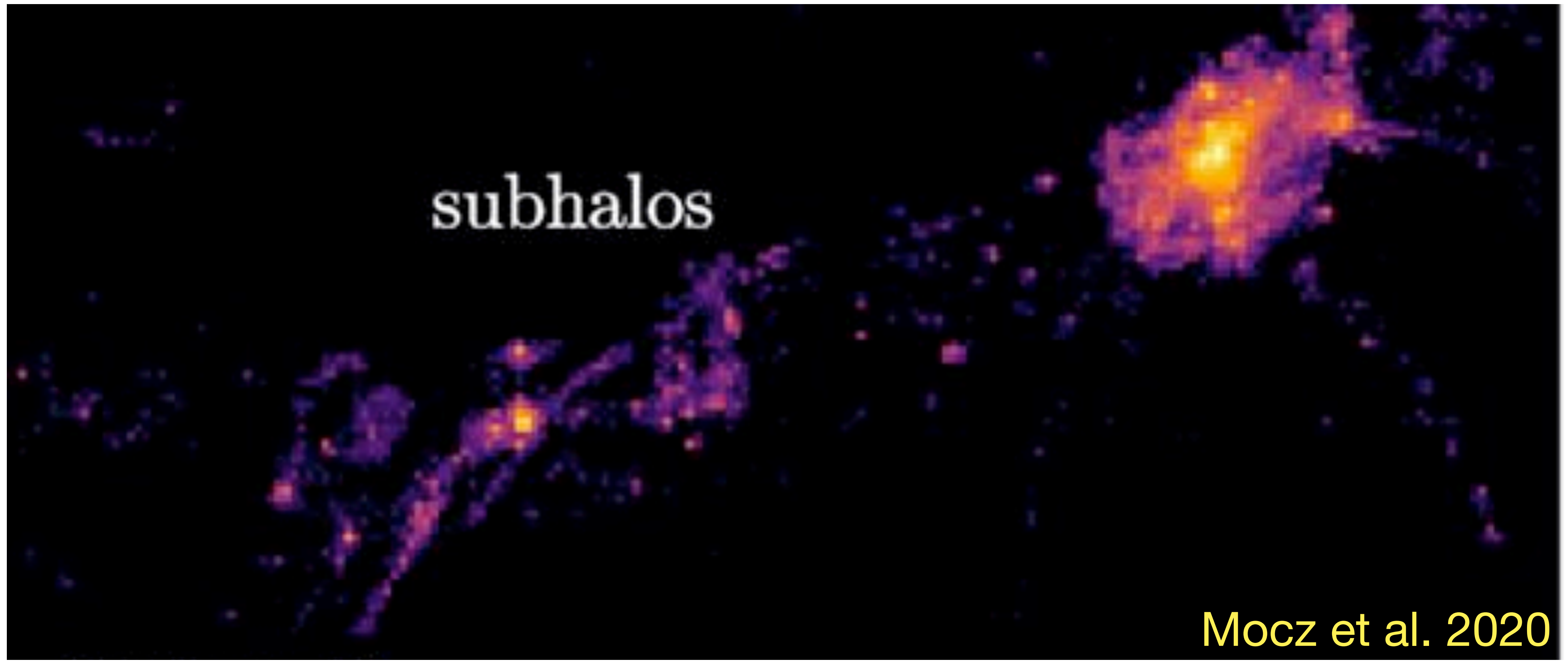
NATURE OF DARK MATTER



NATURE OF DARK MATTER

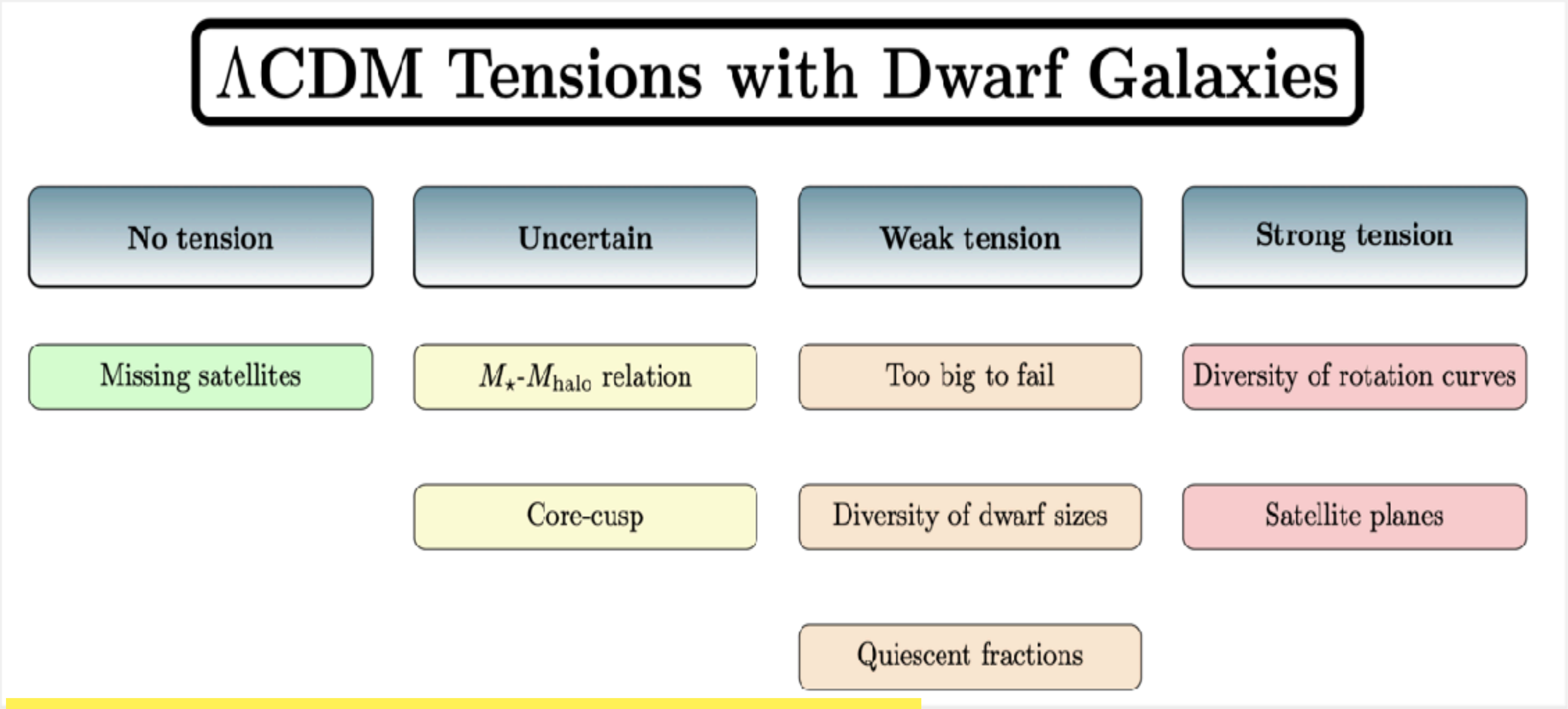


Cold Dark Matter (CDM): dark matter is a cold, collisonless fluid



PROBLEMS WITH COLD DARK MATTER

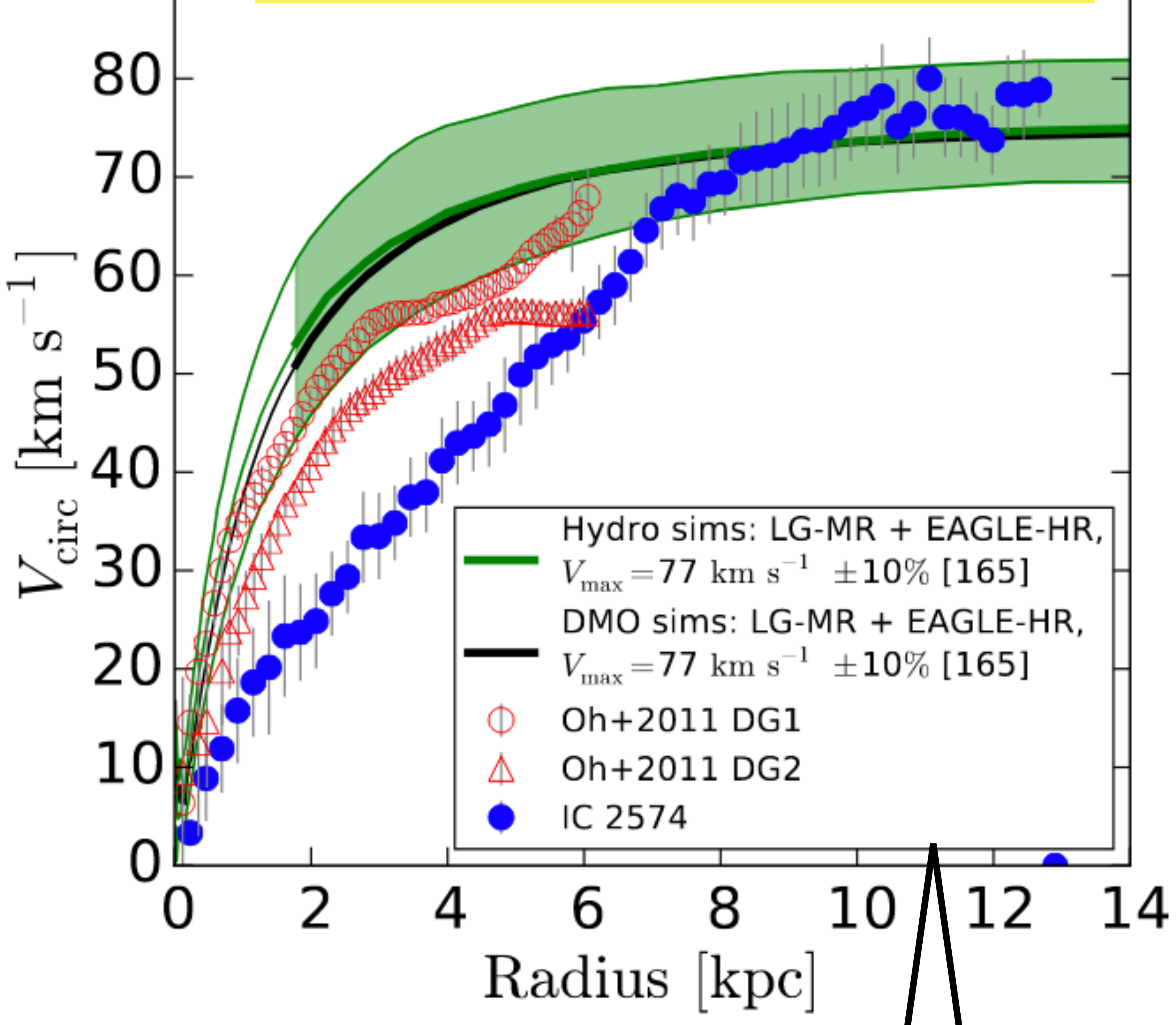
Cold DM Predicts the large scale structure very well, but has some problems at smaller scales



Bullock & Boylan-Kolchin (2017) arXiv:1707.04256

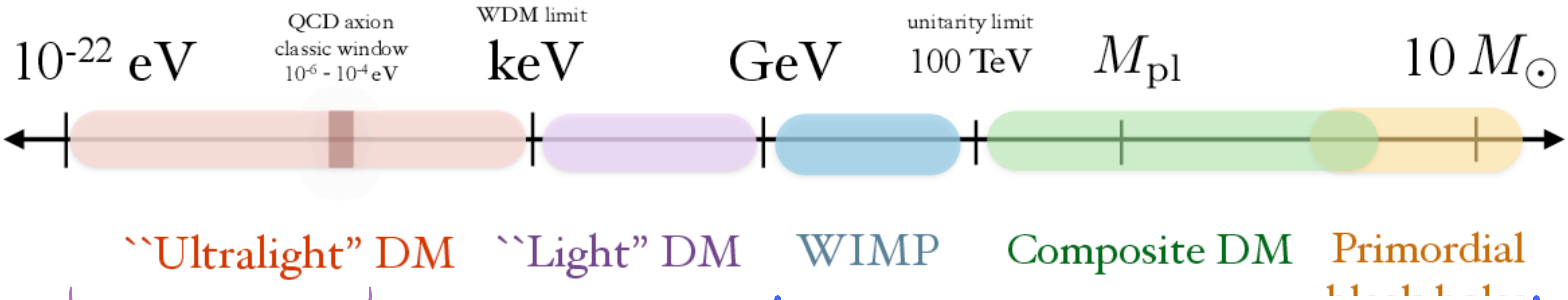
Sales et al (2022) arXiv:2206.05295

Tulin & Yu (2017) arXiv:1705.02358



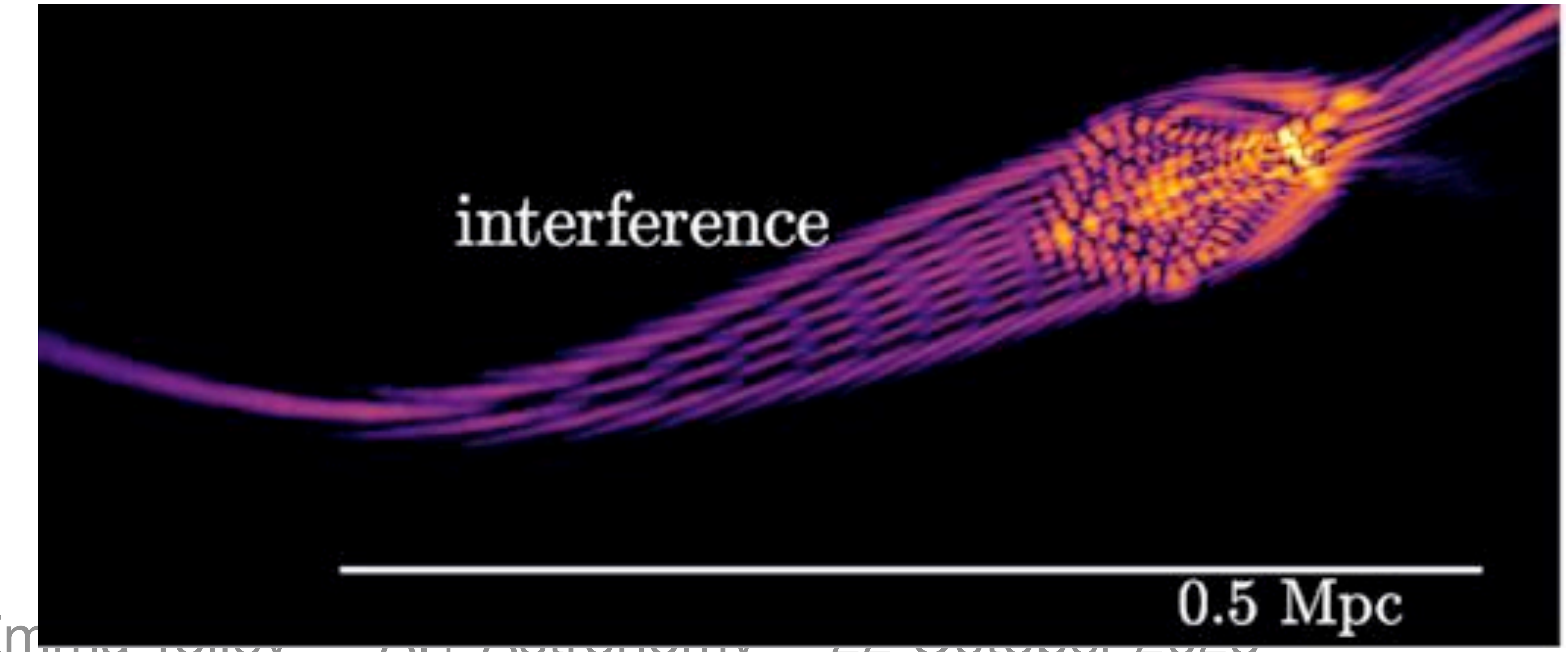
Dwarf galaxy which does not match simulation with baryonic feedback included

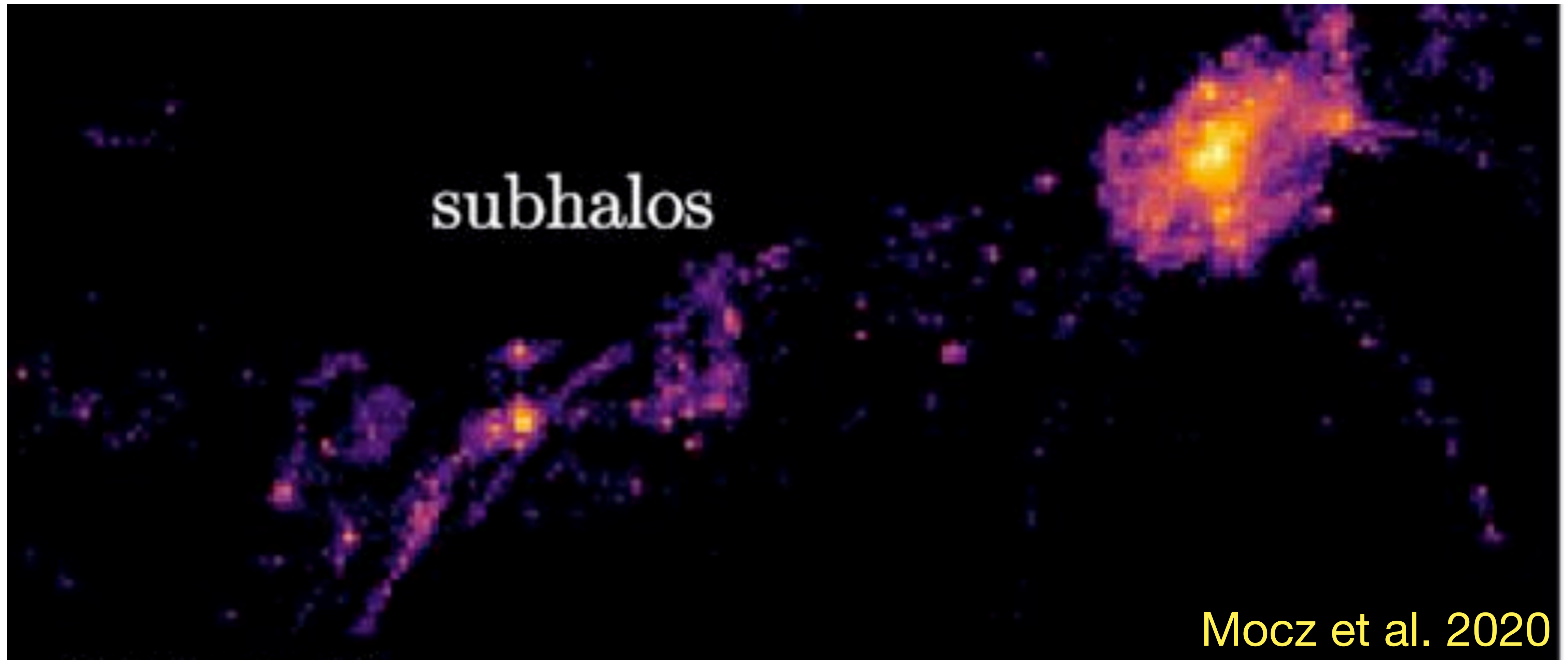
FUZZY VS COLD DARK MATTER



Fuzzy Dark Matter (FDM): dark matter is a quantum wave with a kpc-scale deBroglie wavelength

Cold Dark Matter (CDM): dark matter is a cold, collisonless fluid





FUZZY VS COLD DARK MATTER

FDM obeys the
Shroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

$$\rho \equiv |\psi|^2$$

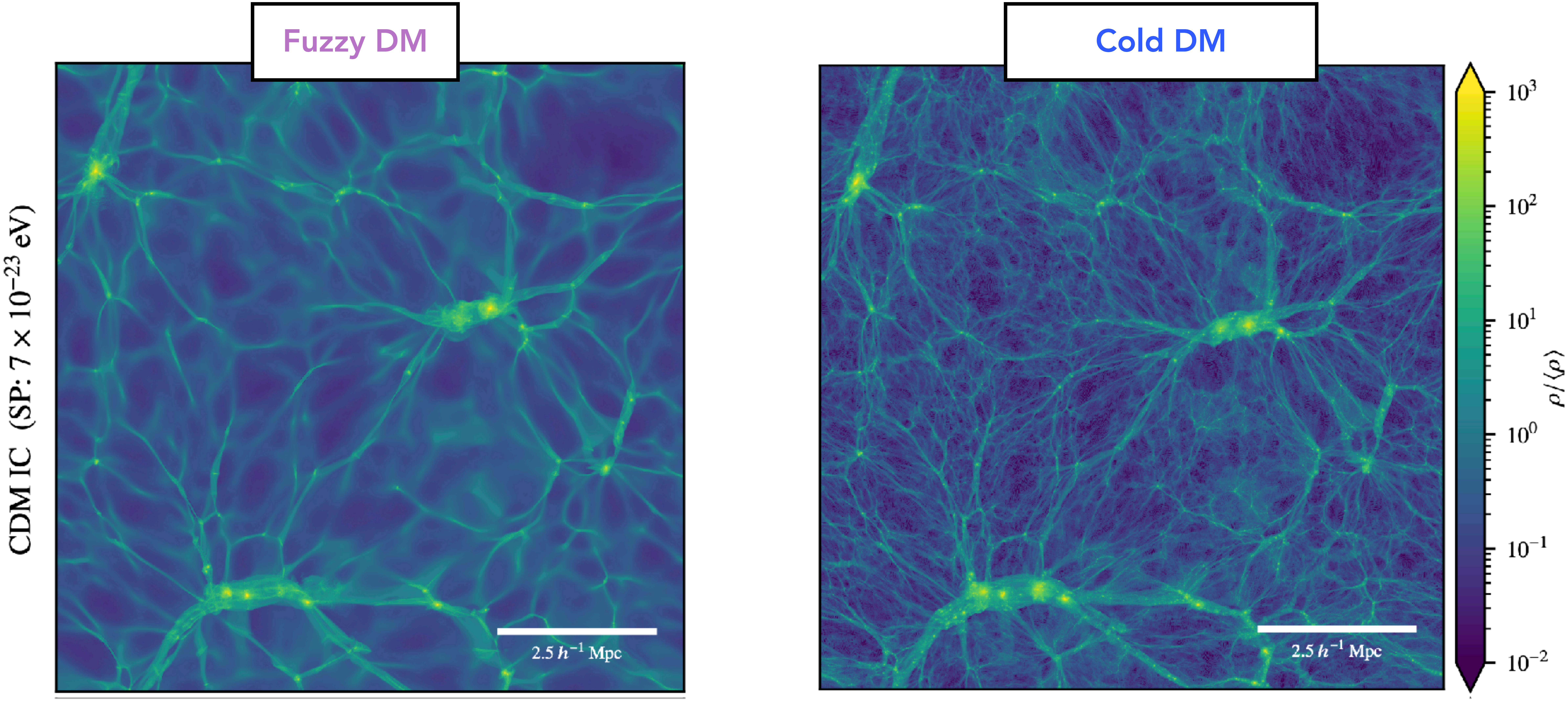
CDM obeys the
Vlasov-Poisson Equations

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} - \nabla V\cdot\frac{\partial f}{\partial \mathbf{v}} = 0,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

$$\rho = \int f\, d^3v$$

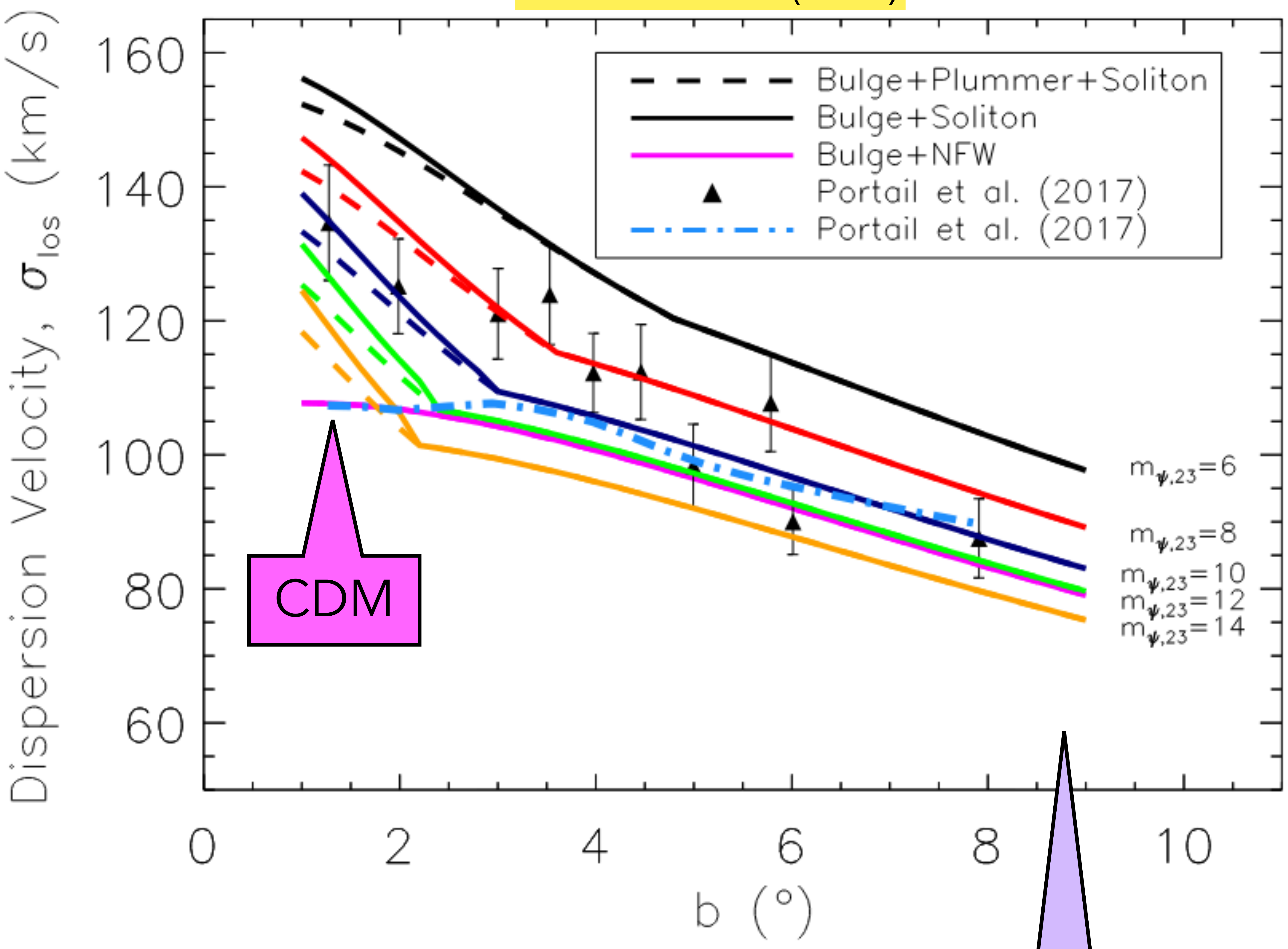
FUZZY VS COLD DARK MATTER



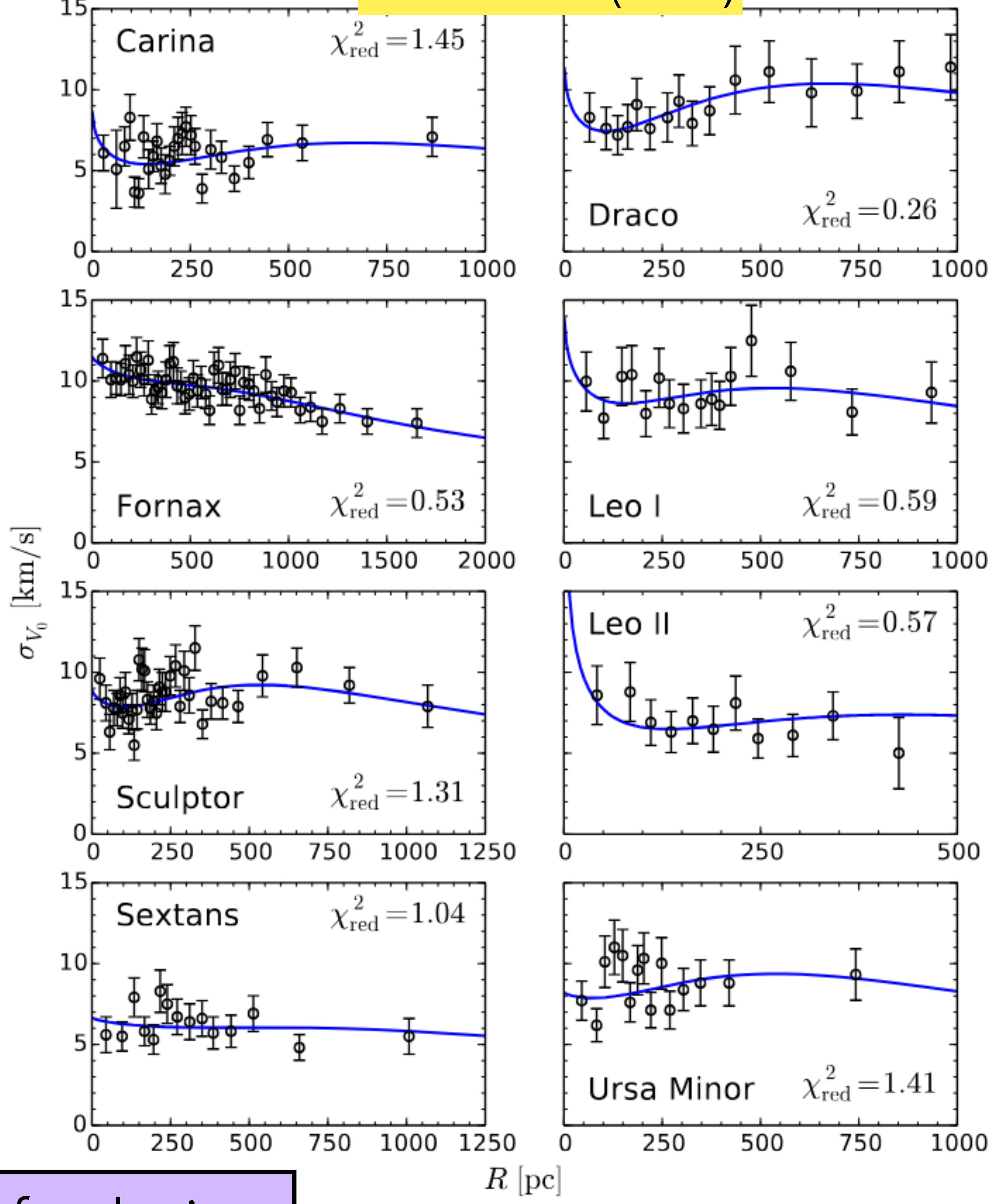
May & Springel (2022), arXiv:2209.14886

FUZZY DARK MATTER

I. De Martino+ (2020)



S-R Chen+ (2016)



FDM can explain halo profiles of our galaxy & many dwarf galaxies

FUZZY DARK MATTER

May & Springel, MNRAS (2021)

Existing simulations very limited in size (**computationally expensive**)

Fuzzy Dark Matter

Schroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

Very **computationally expensive** due to requirement of small step size & fine resolution (need to resolve wave phenomena)

$$\psi \leftarrow e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\Phi}\psi$$ kick

$$\psi \leftarrow \mathrm{IFFT}\left(e^{-i\frac{\hbar}{m}\frac{\Delta t}{2}k^2}\,\mathrm{FFT}(\psi)\right)$$ drift

$$\Phi \leftarrow \mathrm{IFFT}\left(-\frac{1}{k^2}\,\mathrm{FFT}\left(4\pi G m(|\psi|^2 - \langle|\psi|^2\rangle)\right)\right)$$ update potential

$$\psi \leftarrow e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\Phi}\psi$$ kick

Choice of time step: $\Delta t < \min\left(\frac{4}{9\pi}\frac{m}{\hbar}a^2\Delta x^2\ ,\ 2\pi\frac{\hbar}{m}a\frac{1}{|\Phi_{\mathrm{max}}|}\right)$

PHYSICS INFORMED MACHINE LEARNING

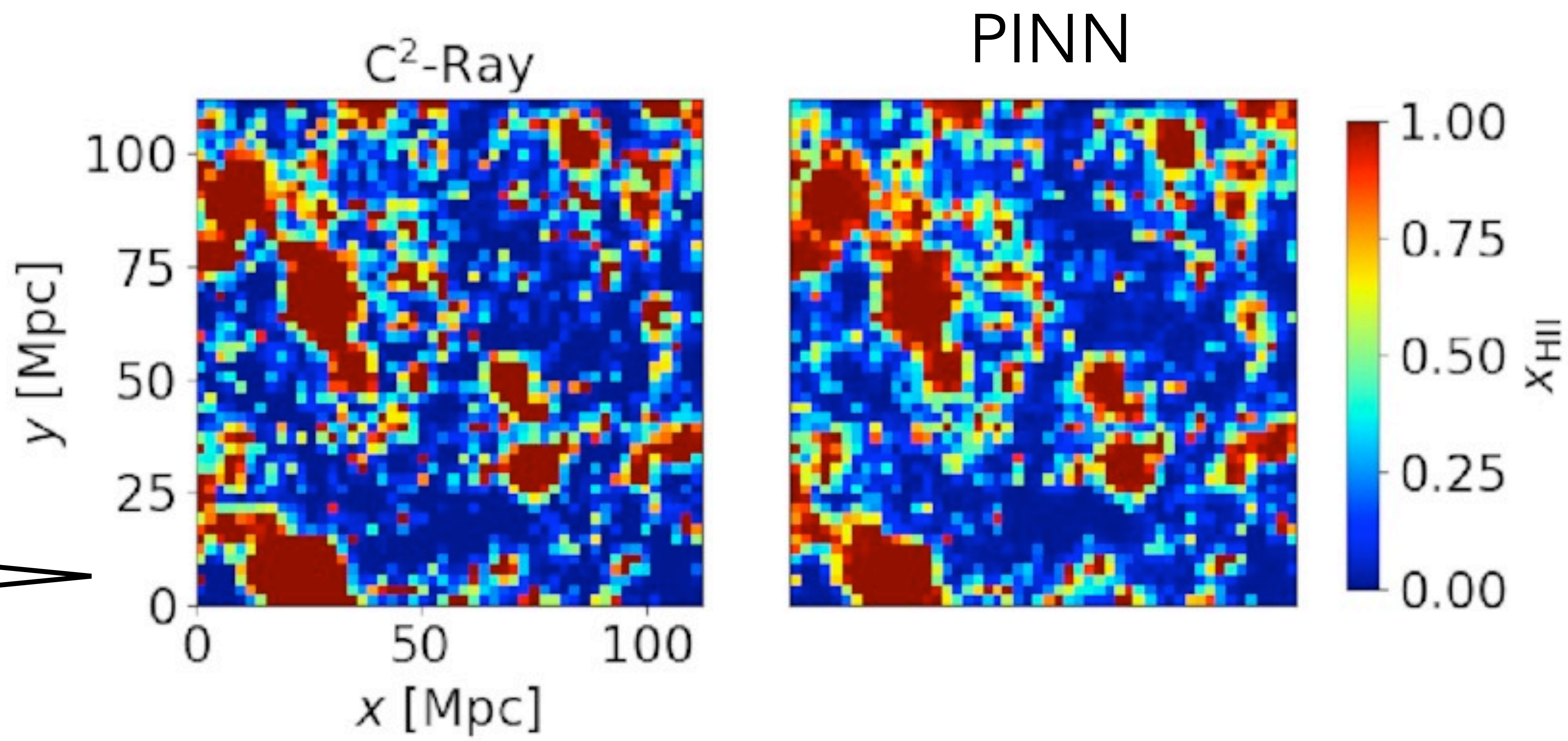
Physics-informed neural networks (PINNs) incorporate physical laws to efficiently solve PDEs

- Can be semi-supervised or **unsupervised** (no training data)
- Can take advantage of **AI hardware** like GPUs, tensor cores, etc
- Can easily **validate** network outputs using the PDE loss

Semi-supervised PINN can predict entire 4D reionization history with only 5 snapshots from true simulation and the constraint:

$$\frac{dx_{\mathrm{HII}}}{dt} = (1 - x_{\mathrm{HII}})\Gamma - C\alpha_{\mathrm{B}} n_{\mathrm{H}} x_{\mathrm{HII}}^2$$

Training time: **1.5 hours** on one NVIDIA Tesla P100 16GB GPU



Korber, Bianco, Tolley, Kneib. MNRAS (2023) DOI: 10.1093/mnras/stad615

PHYSICS INFORMED MACHINE LEARNING

Schroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

$$\rho \equiv |\psi|^2$$

Can we develop a PINN to solve for ψ?

PHYSICS INFORMED MACHINE LEARNING

Schroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \bar{\rho}),$$

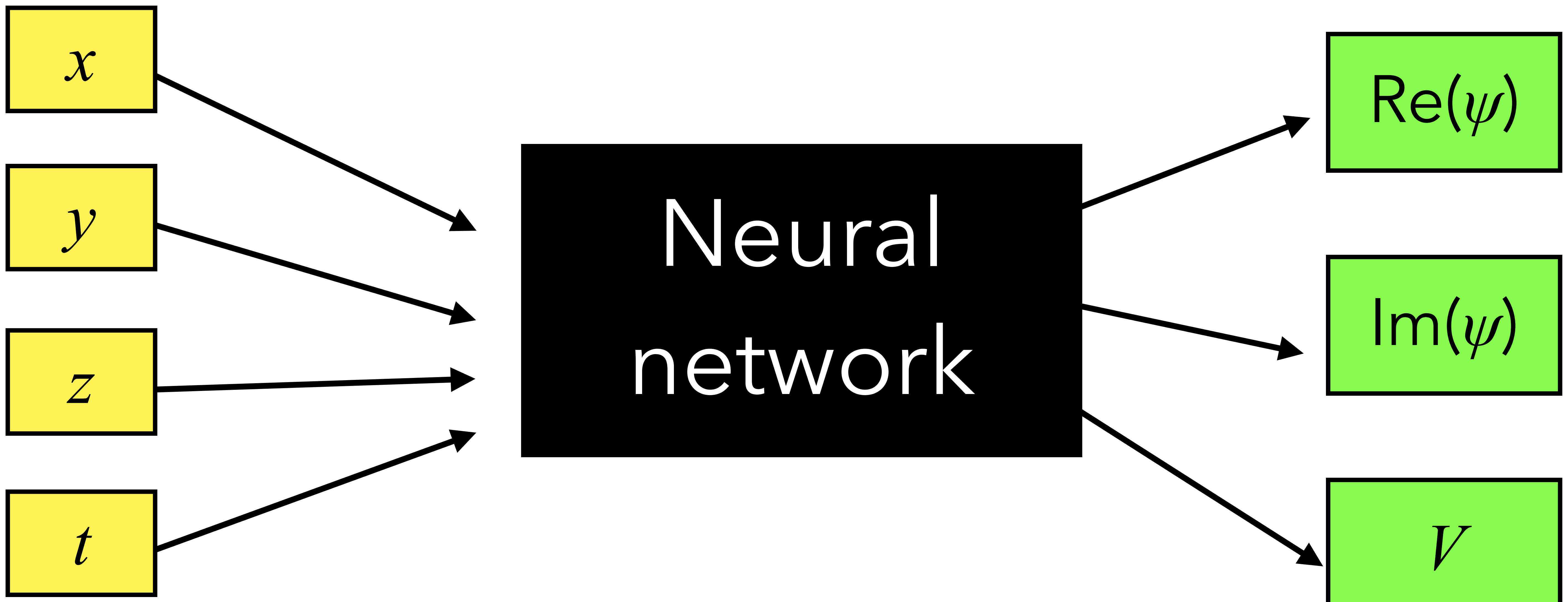
$$\rho \equiv |\psi|^2$$

Theorem (*Cybenko, 1989*)

Let σ be any continuous sigmoidal function. Then, the finite sums of the form

$$g(x) = \sum_{j=1}^{N} w_j^2 \sigma((w_j^1)^T x + b_j^1)$$

are dense in $C(I_d)$.



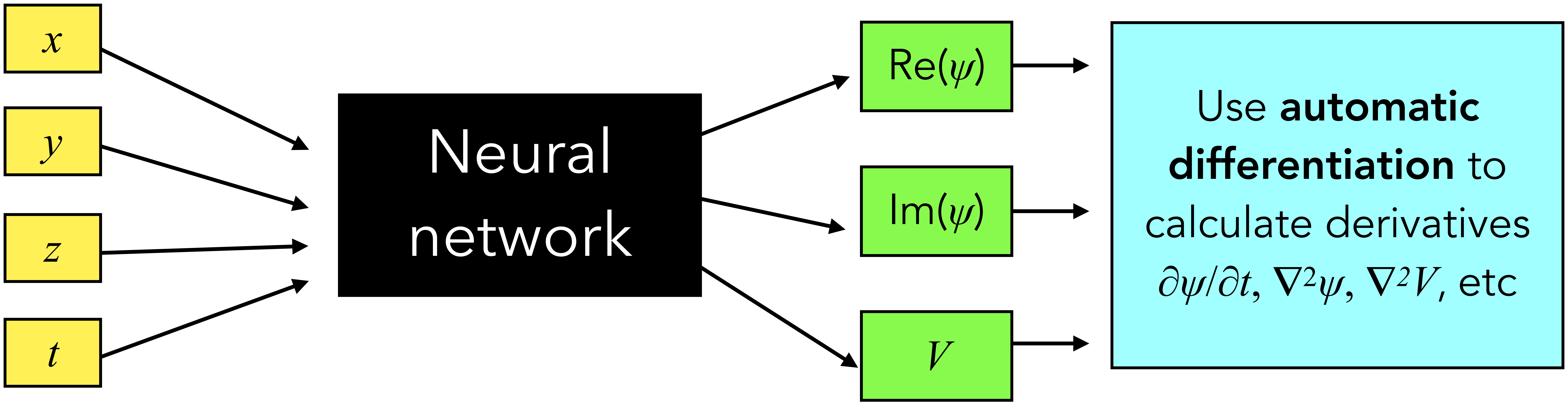
Physics Informed Machine Learning

Schroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

$$\rho \equiv |\psi|^2$$



PHYSICS INFORMED MACHINE LEARNING

Schroedinger-Poisson Equations

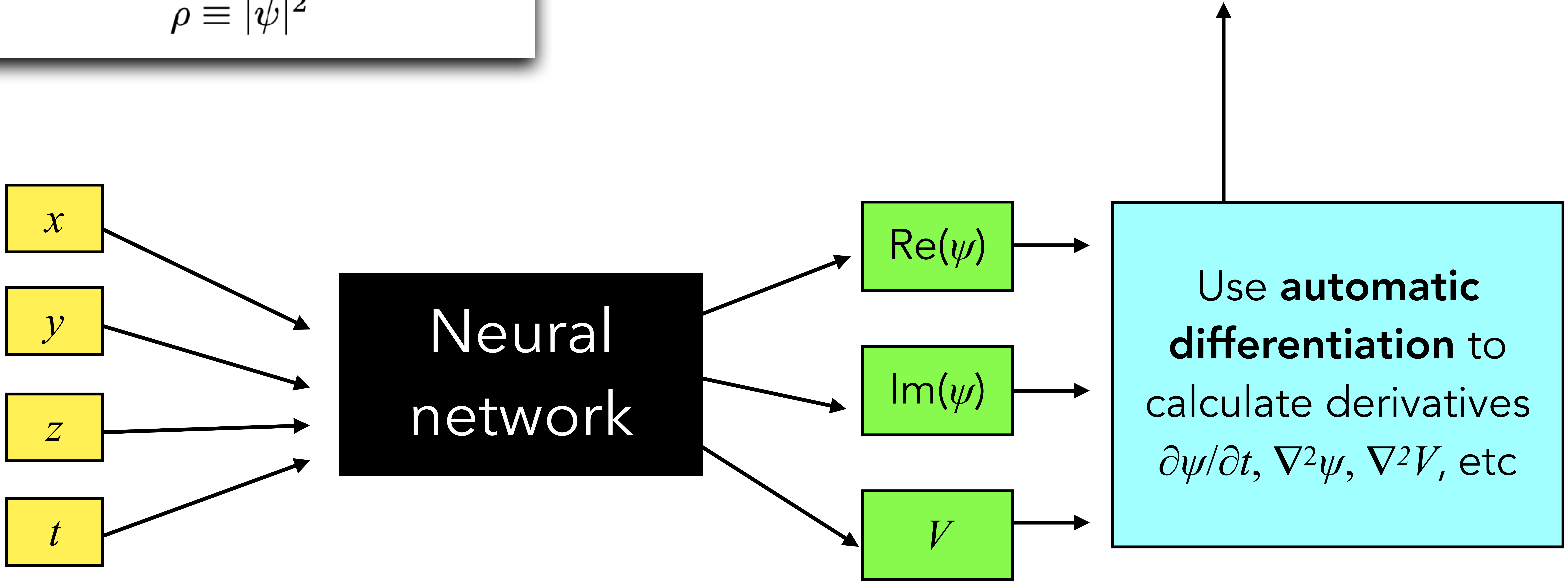
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \overline{\rho}),$$

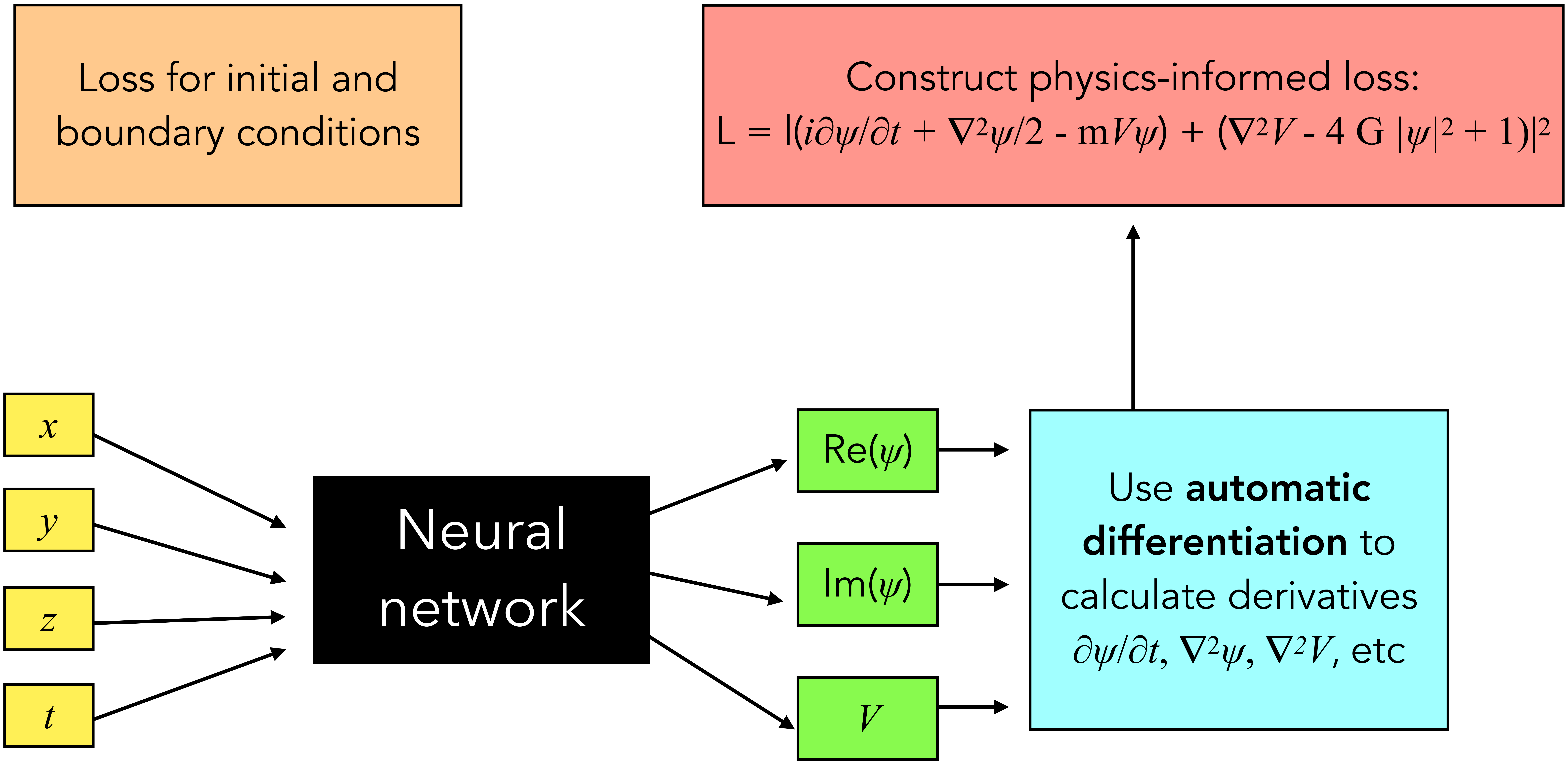
$$\rho \equiv |\psi|^2$$

Construct physics-informed loss:

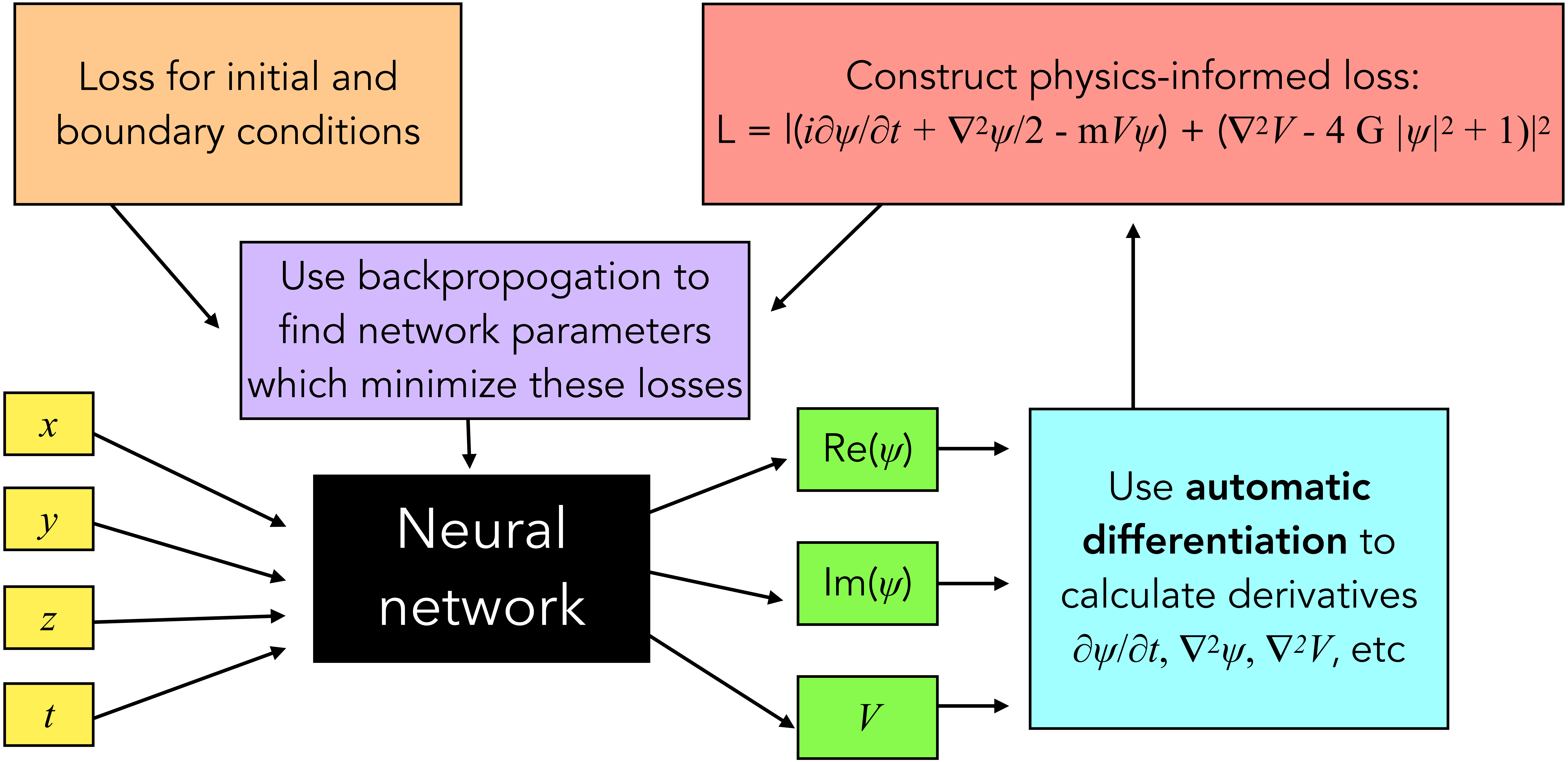
L = $|(i\partial\psi/\partial t + \nabla^2\psi/2 - mV\psi) + (\nabla^2 V - 4\,G\,|\psi|^2 + 1)|^2$



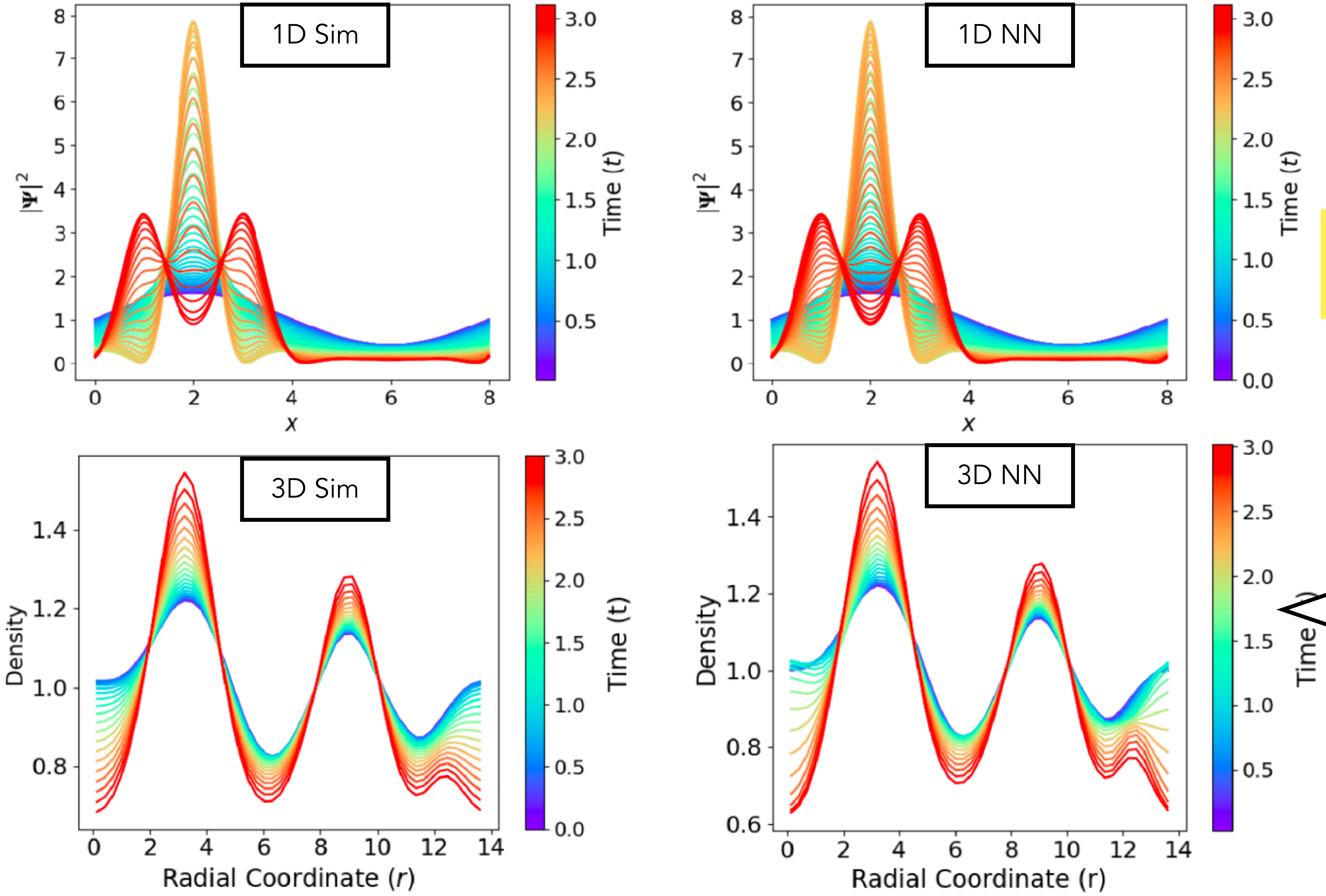
PHYSICS INFORMED MACHINE LEARNING



PHYSICS INFORMED MACHINE LEARNING



SOLVING FUZZY DARK MATTER

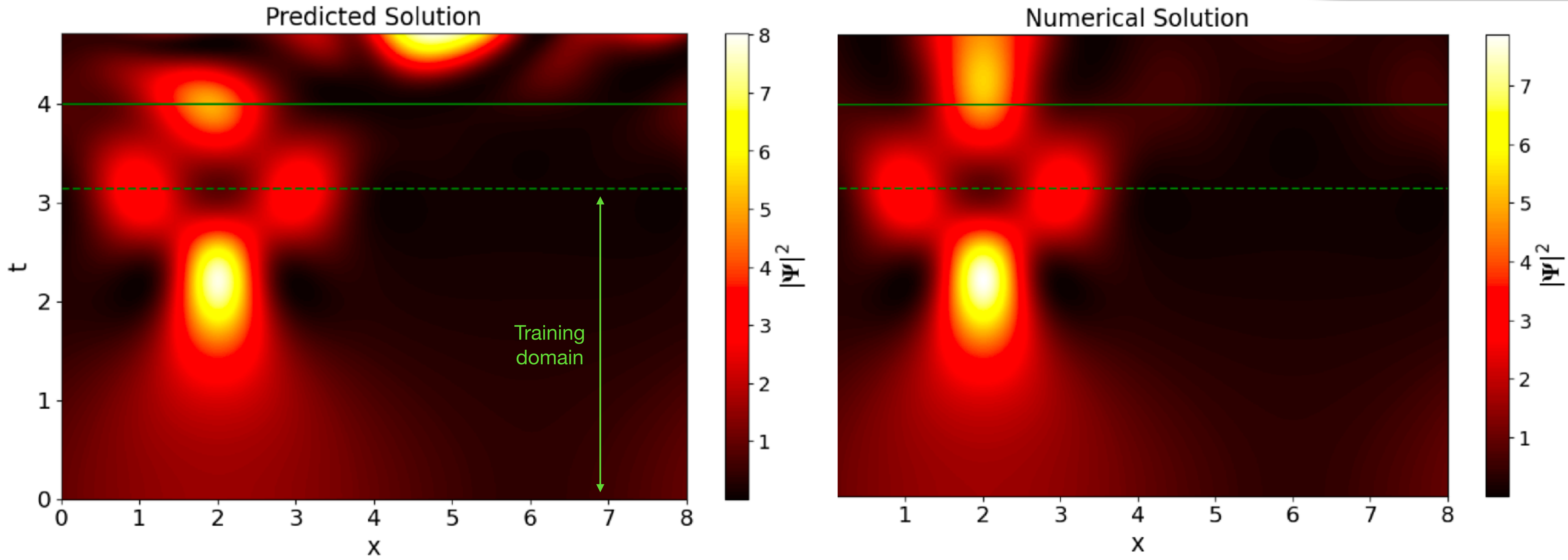


Mishra & Tolley 2025, *ApJ* **988** 114

Unsupervised neural network predicting Fuzzy DM dynamics using only physics constraints and initial conditions

SOLVING FUZZY DARK MATTER

Mishra & Tolley 2025,
ApJ **988** 114

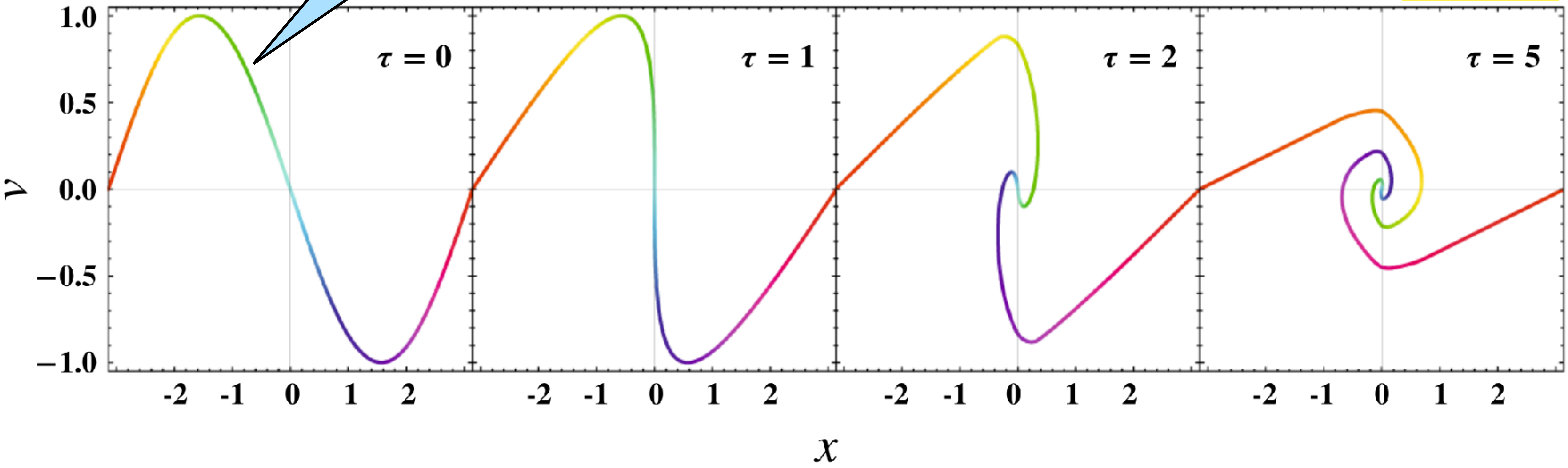



WHAT ABOUT CDM?

Cold Dark Matter

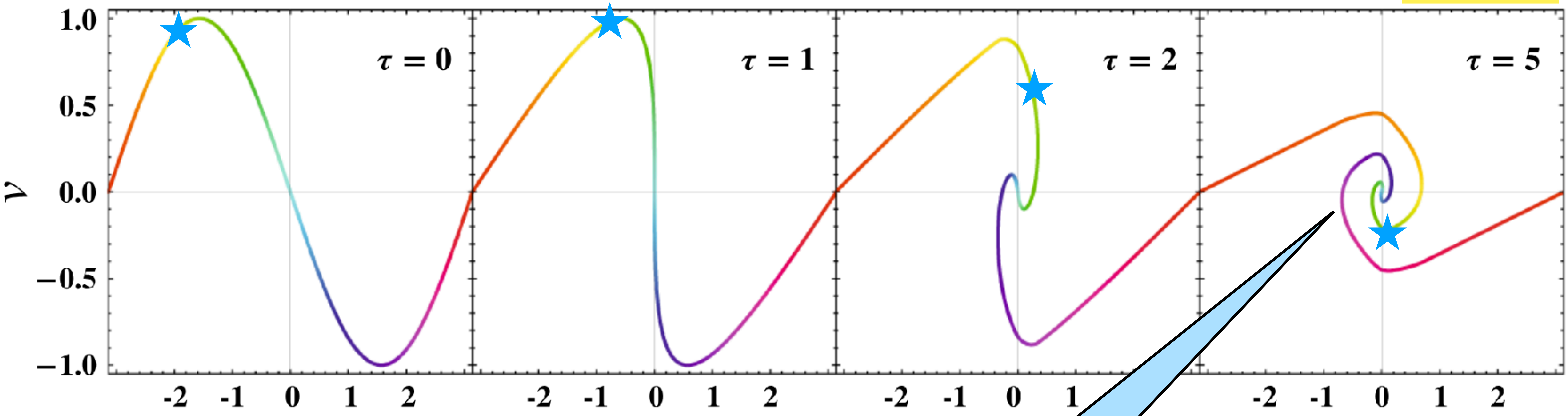
Cold: vanishing velocity dispersion

Rampf 2021



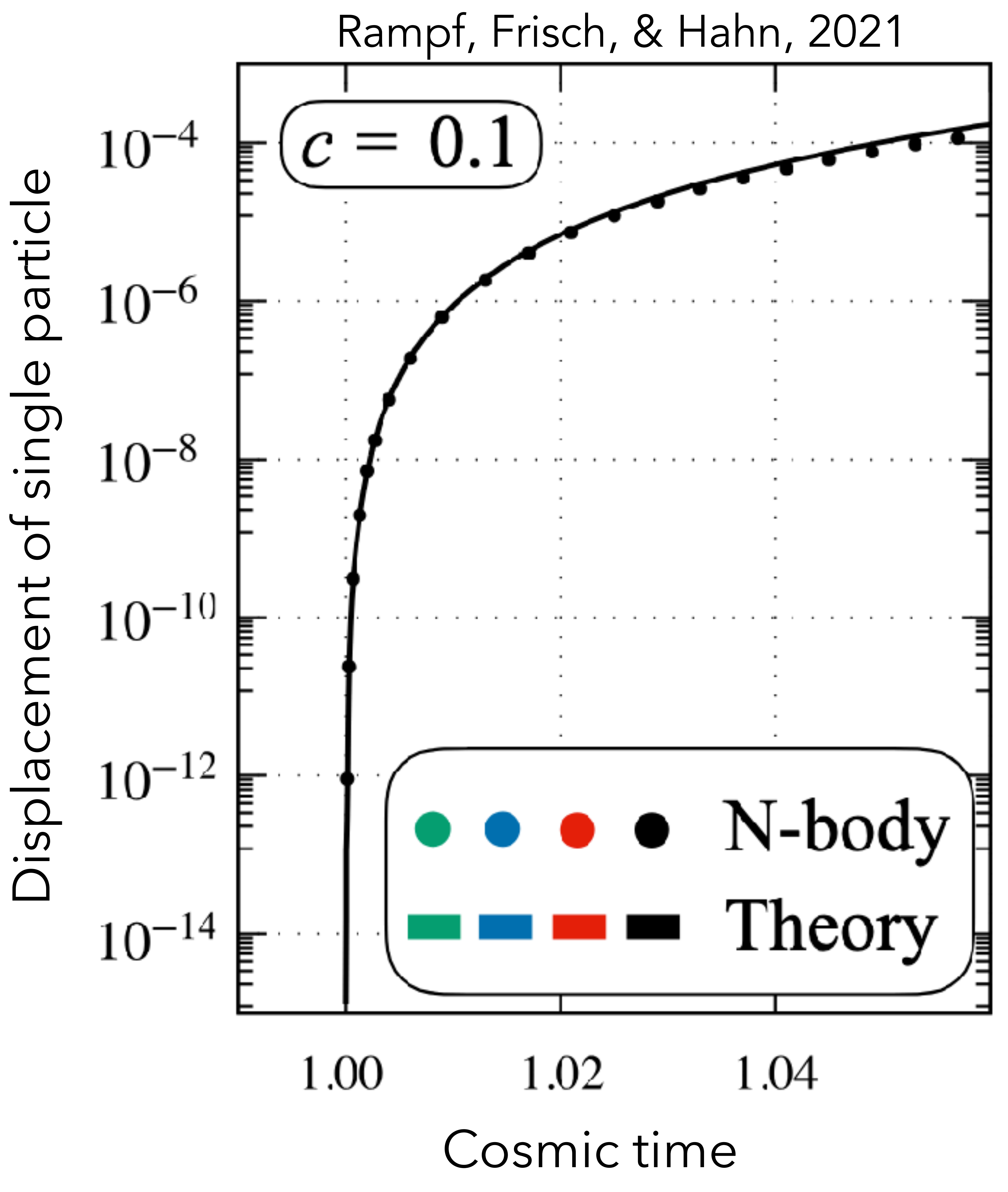
COLD DARK MATTER

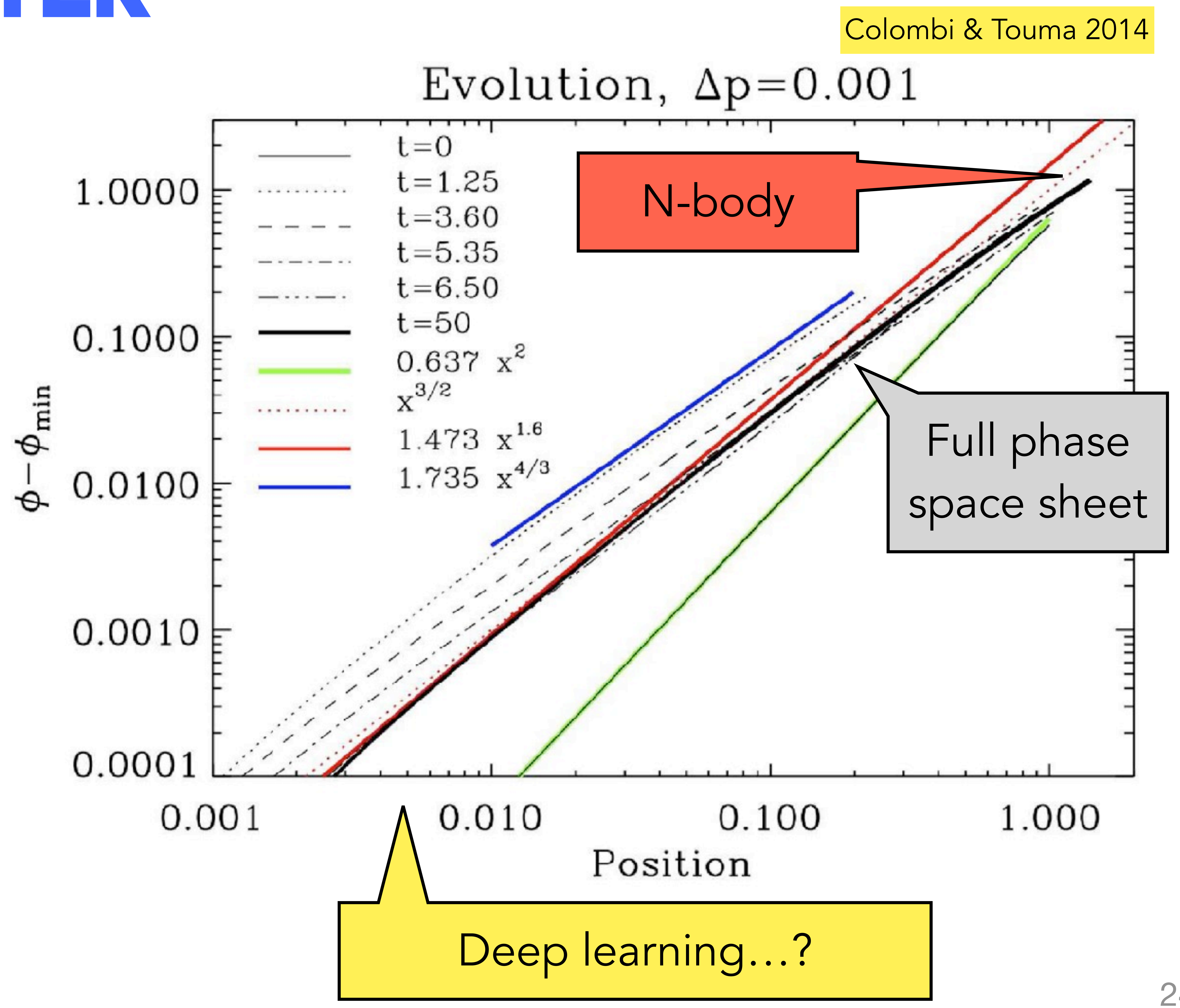
Rampf 2021



N-body simulation evolves points along this sheet in phase space

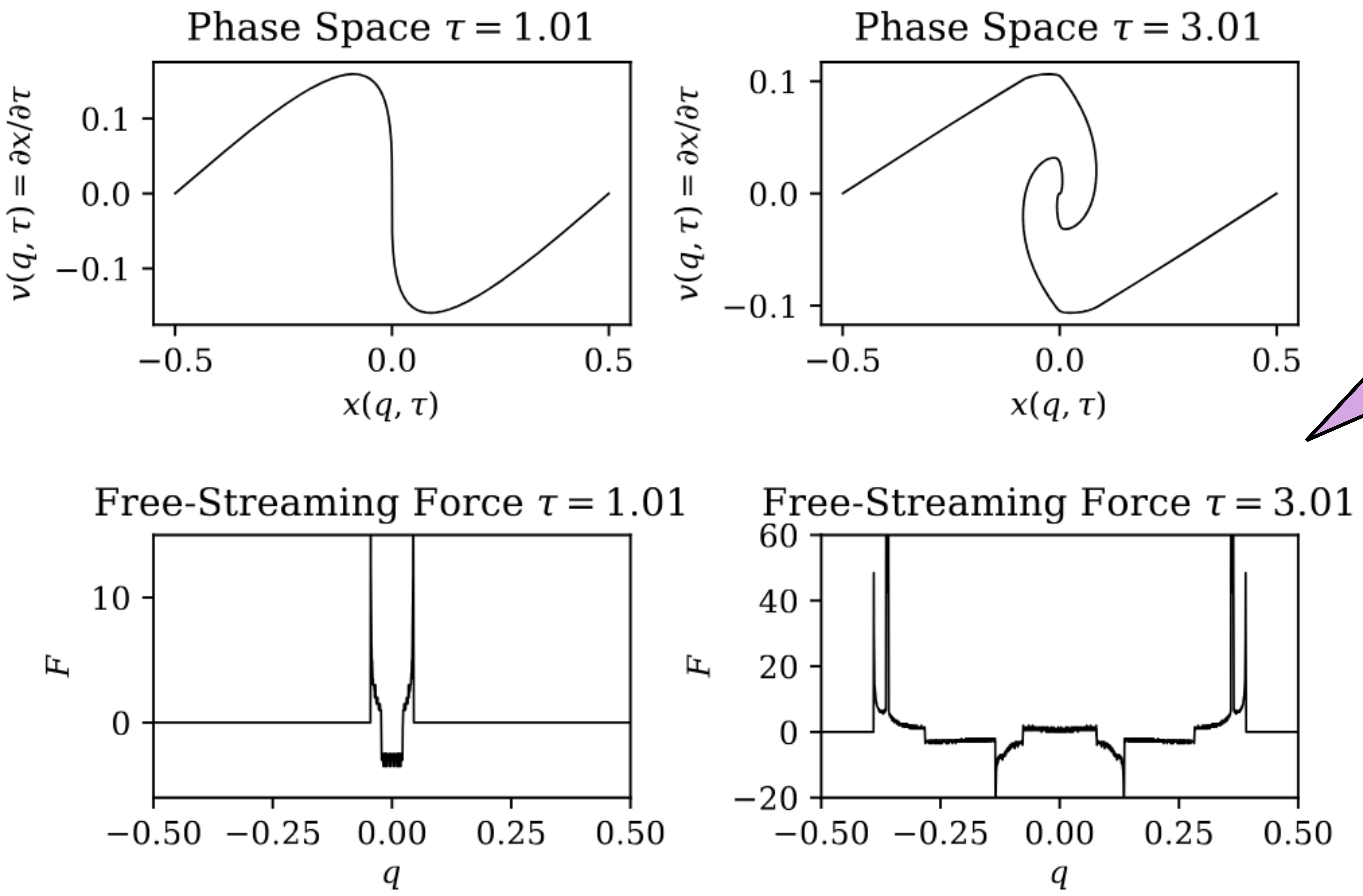
COLD DARK MATTER



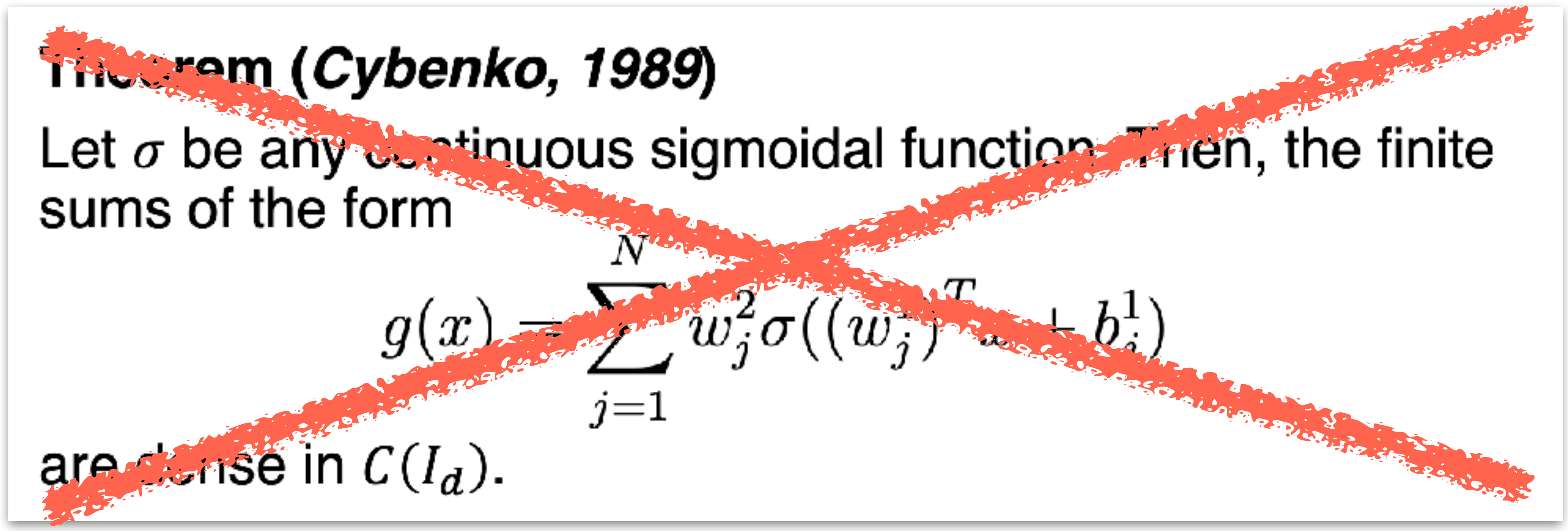


COLD DARK MATTER

Cerardi, Tolley, Mishra, submitted to MNRAS



Discontinuous force -> discontinuous acceleration, very difficult to model with traditional neural network


Theorem (***Cybenko, 1989***)
Let σ be any continuous sigmoidal function. Then, the finite sums of the form

$$g(x) = \sum_{j=1}^{N} w_j^2 \sigma((w_j^1)^T x + b_j^1)$$

are dense in $C(I_d)$.

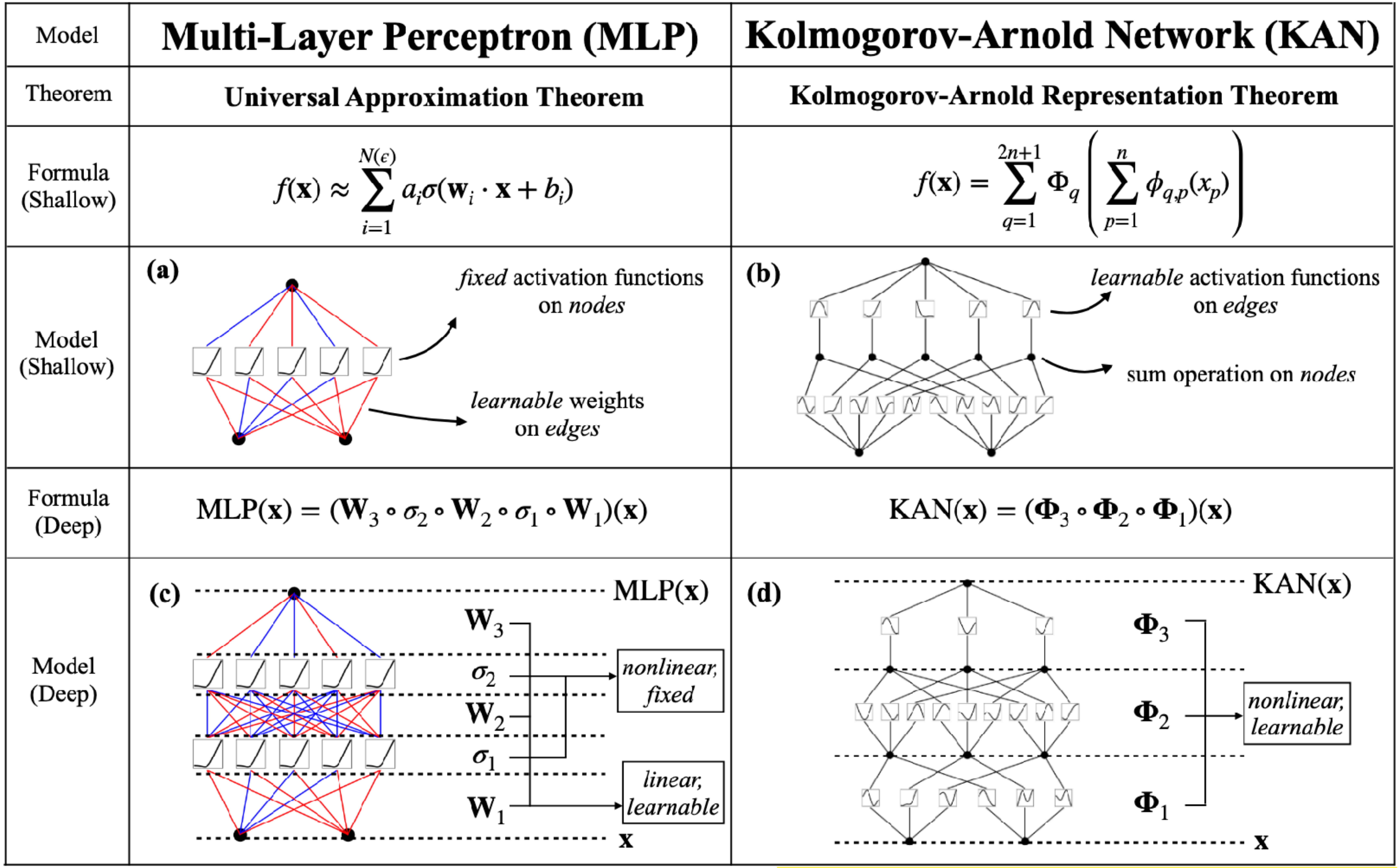
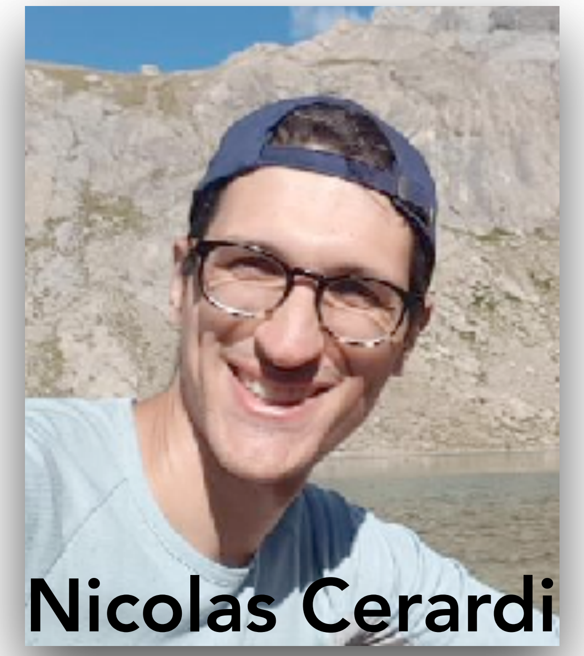


Figure from Liu et al (2024), arXiv:2404.19756

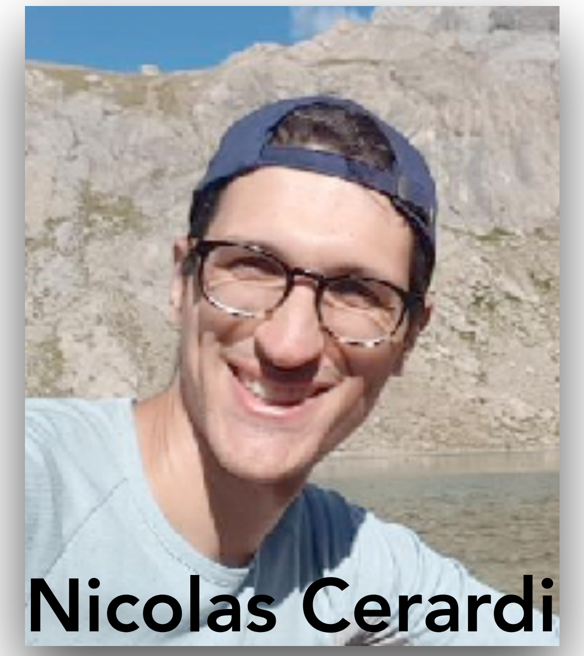
CDM PINN

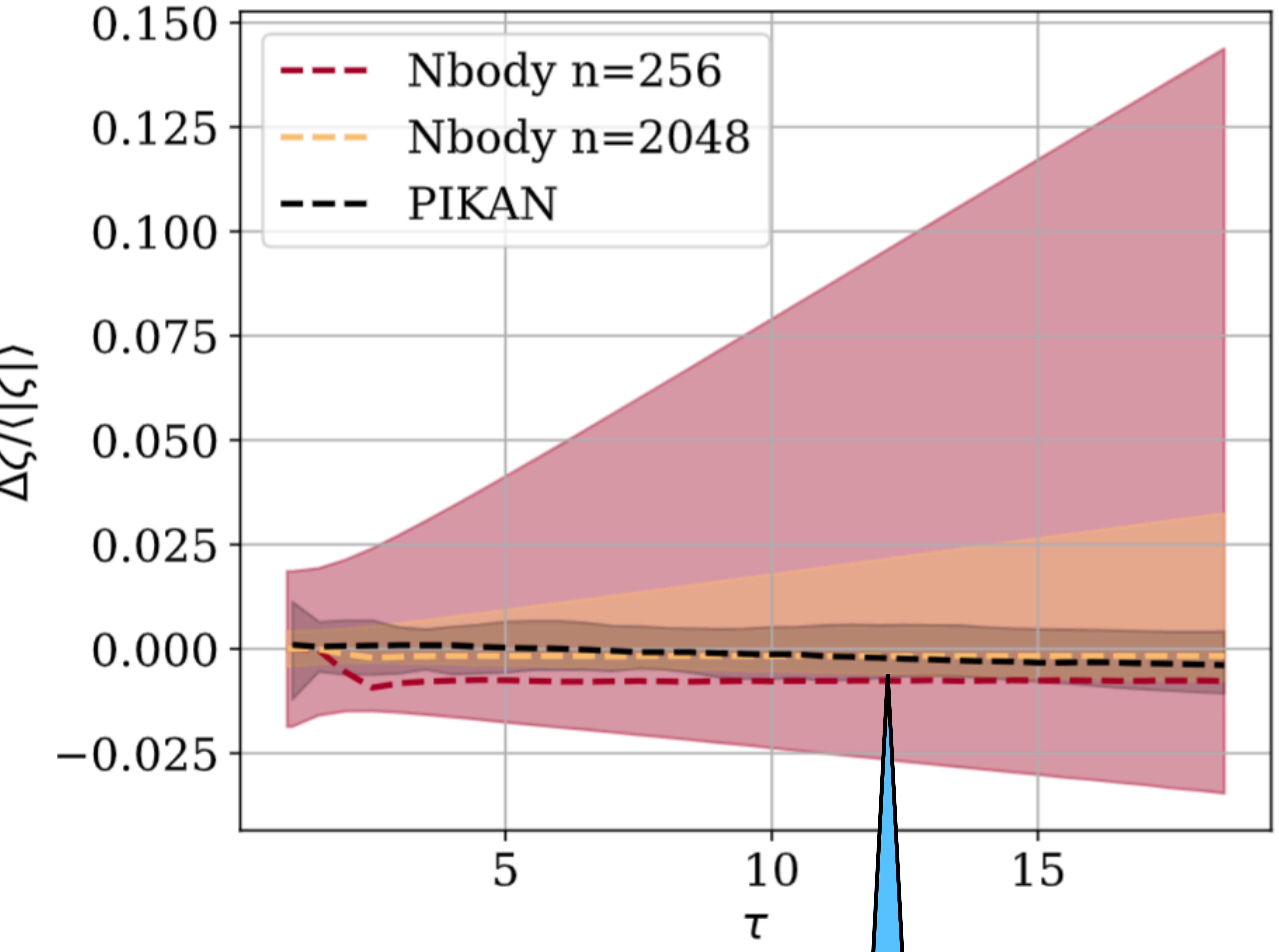
Cerardi, Tolley, Mishra, submitted to MNRAS

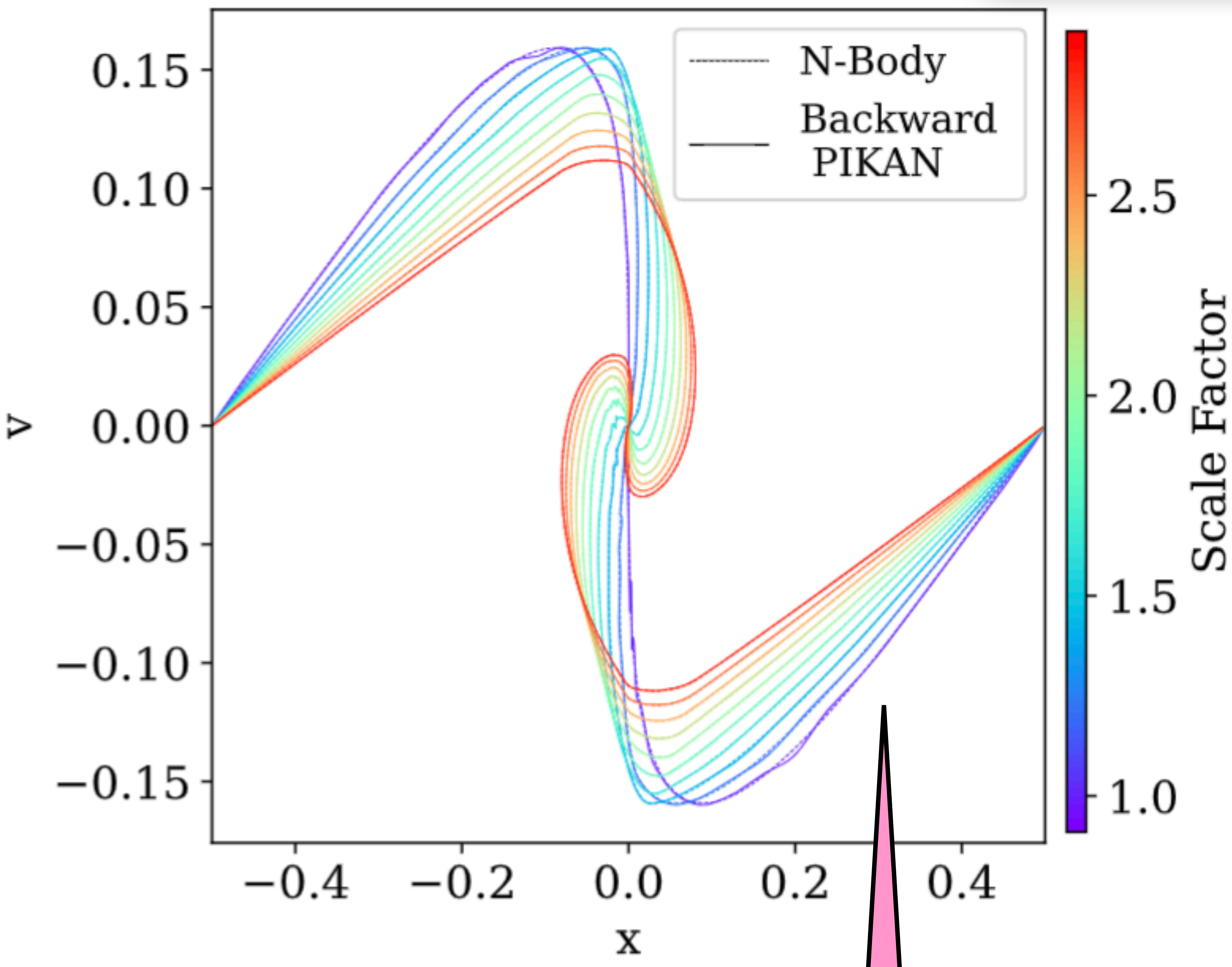


CDM PINN

Cerardi, Tolley, Mishra, submitted to MNRAS







Much smaller errors in PINN

Can also evolve simulation backwards in time

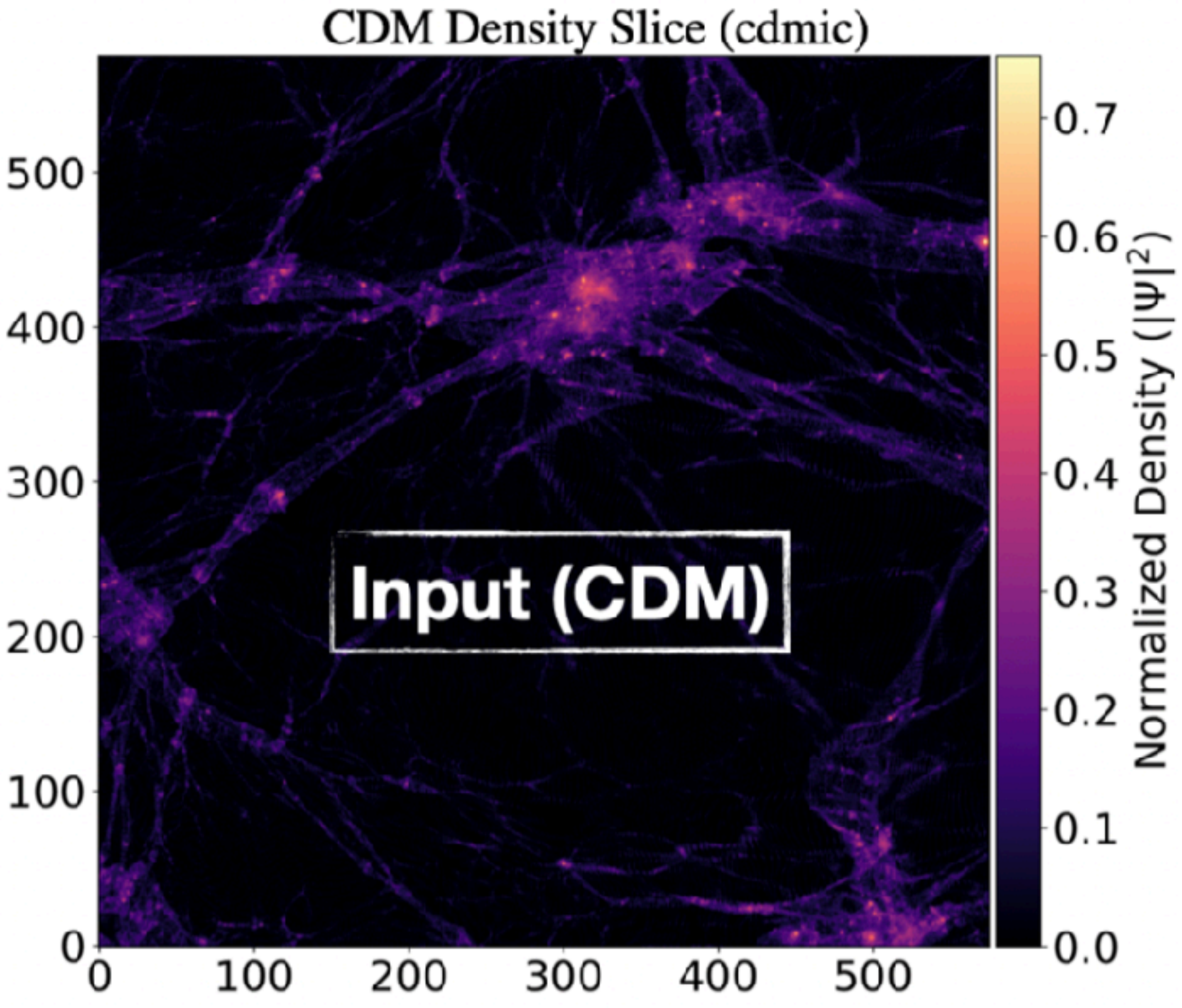
...IN SUMMARY

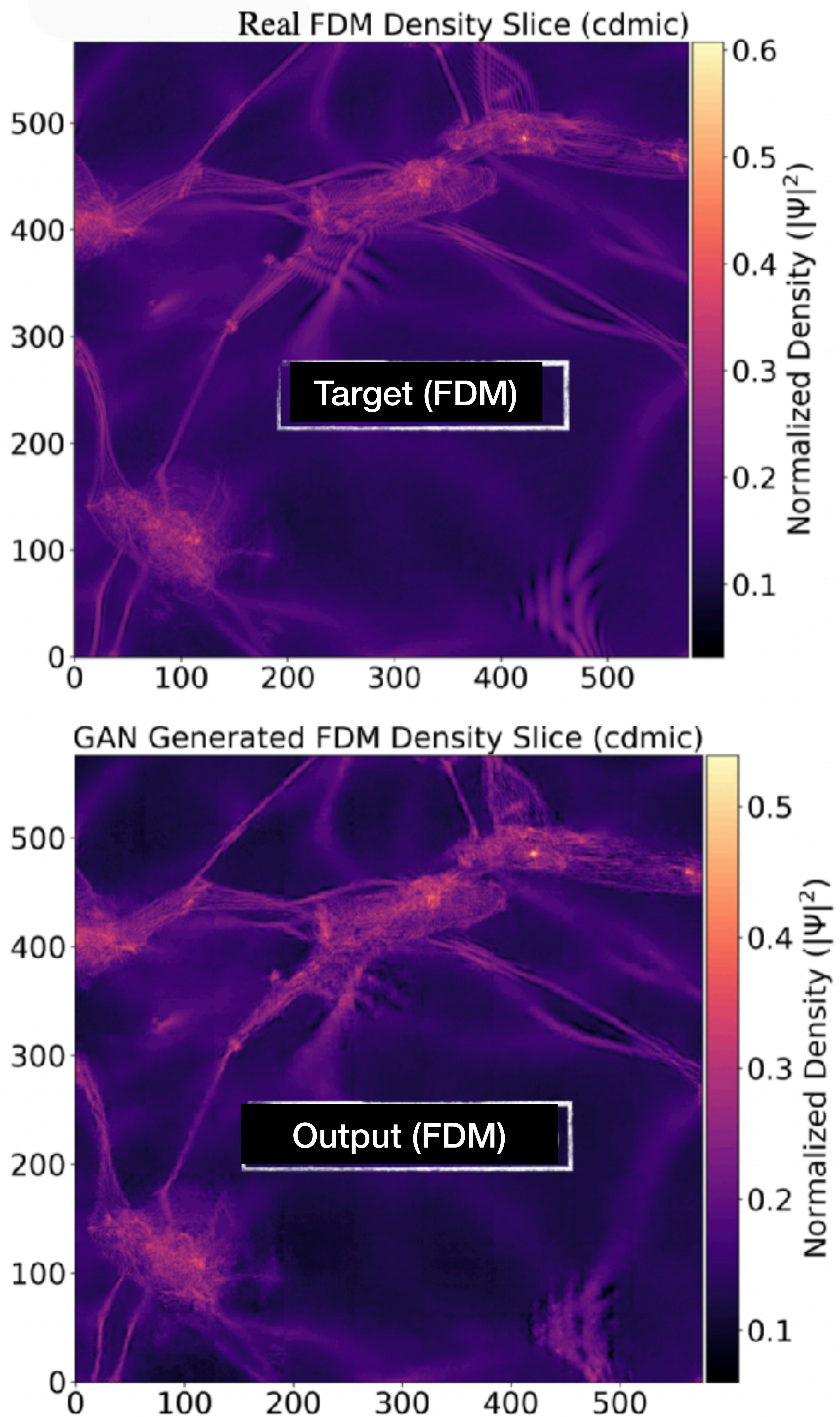
- **Physics-informed neural networks as PDE solvers for cosmology**
  - **Challenges**: Extremely **large space/time domains** needed for cosmology, and long-ranged **gravitational force** needs to be calculated across entire spatial domain
- Exploring physics informed neural networks to solve PDEs for **CDM** (1D only) and **FDM** (1D & 3D)
  - Initial results show excellent results, including **better error accumulation** compared to numerical solvers
  - But so far **not computationally cheaper** compared to traditional methods
    - Exploring implementations with NVIDIA PhysicsNeMo to improve performance

...NEXT STEPS

Now exploring **hybrid methods**, conditional generative models with physics constraints

Plots by A. Mishra

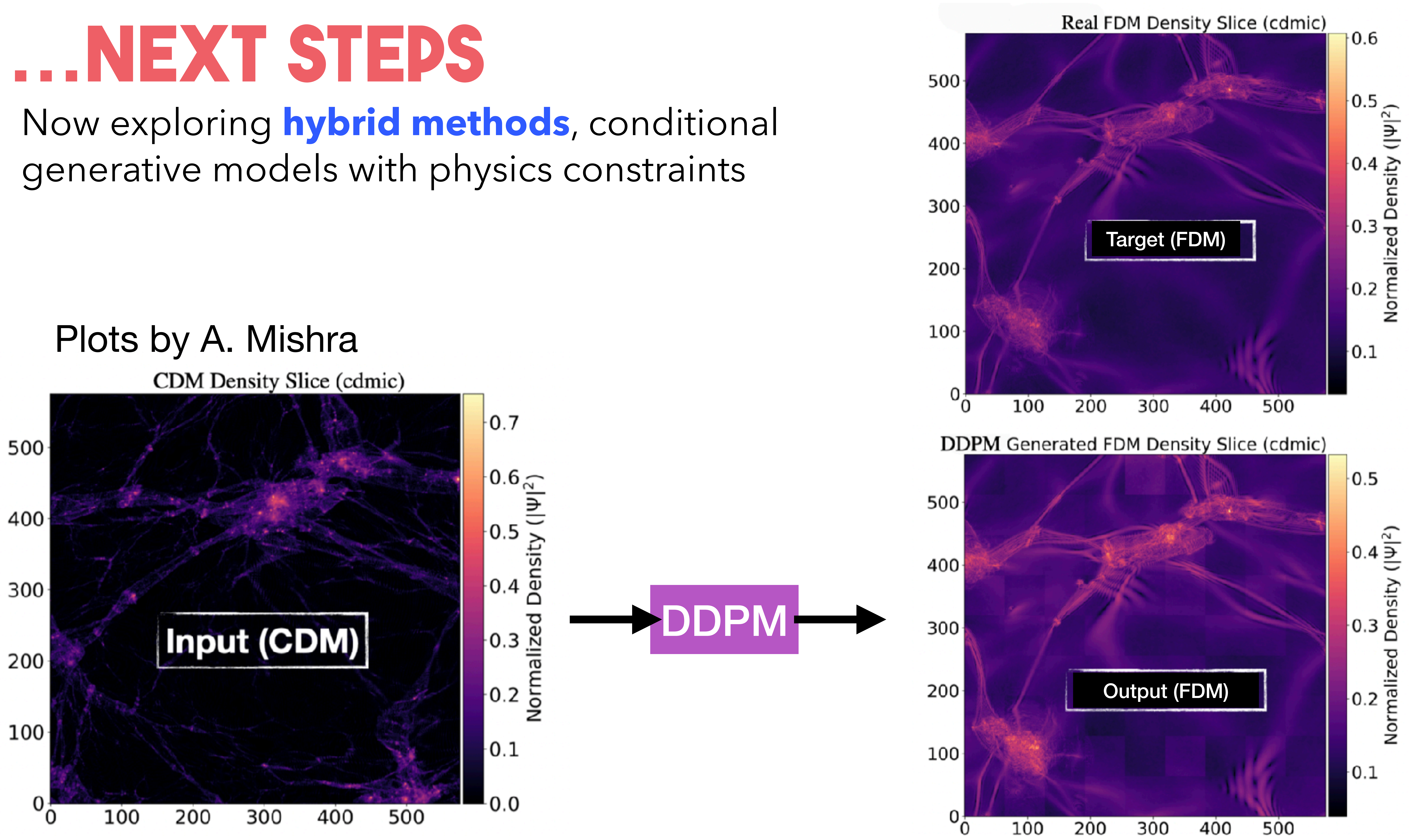




...NEXT STEPS

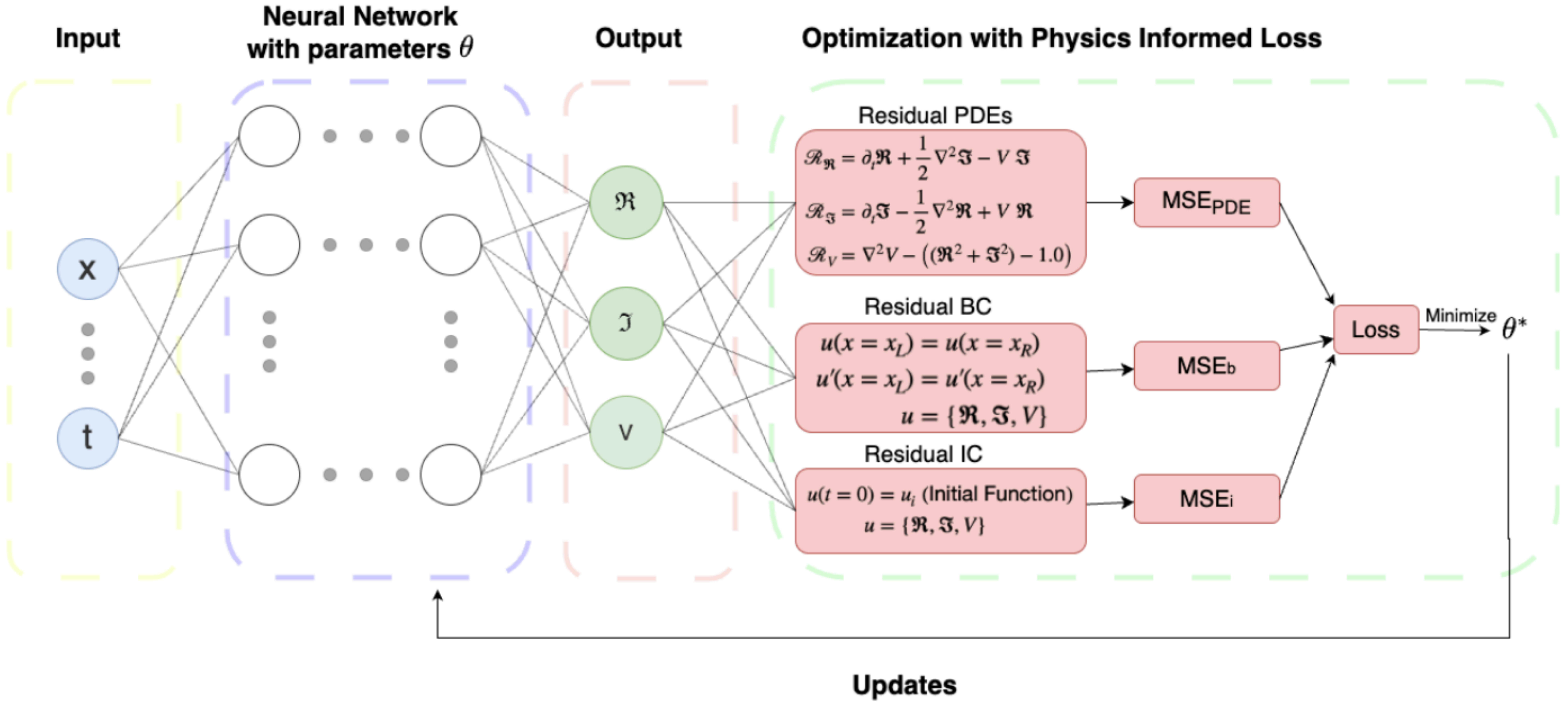
Now exploring **hybrid methods**, conditional generative models with physics constraints

Plots by A. Mishra



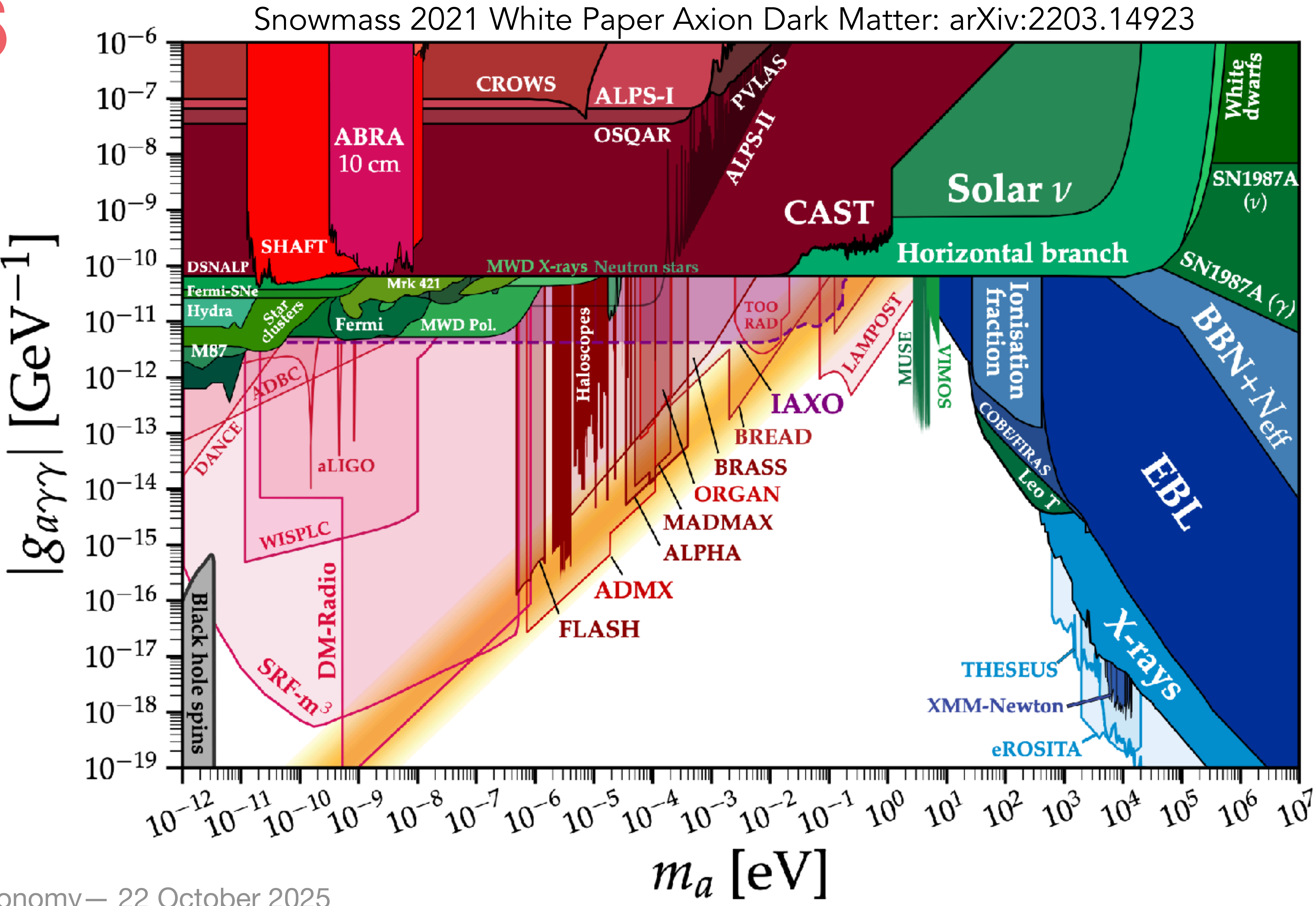

THANK YOU!

PHYSICS-INFORMED DEEP LEARNING



Mishra & Tolley, submitted to APJ

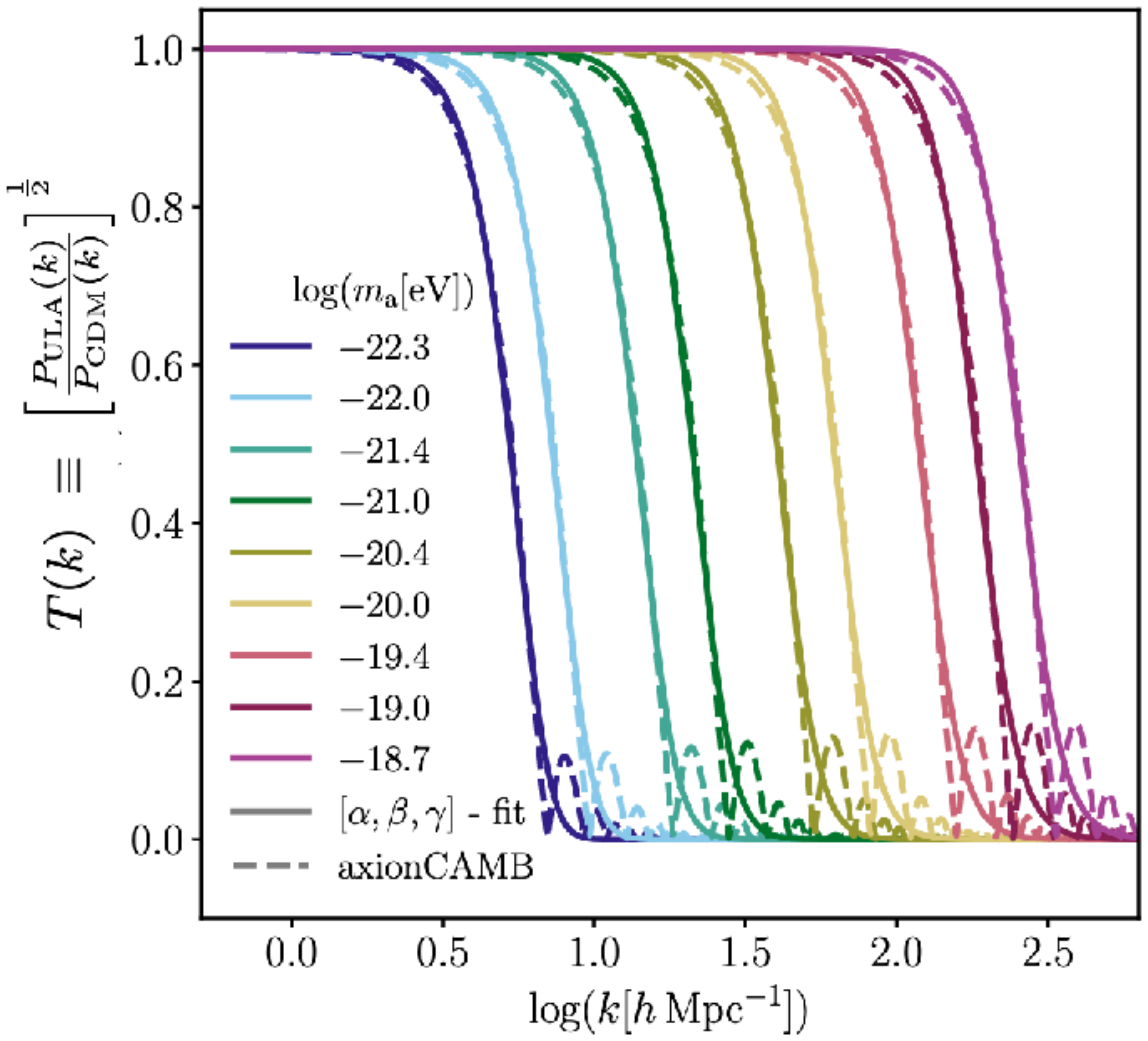
AXIONS & ALPS

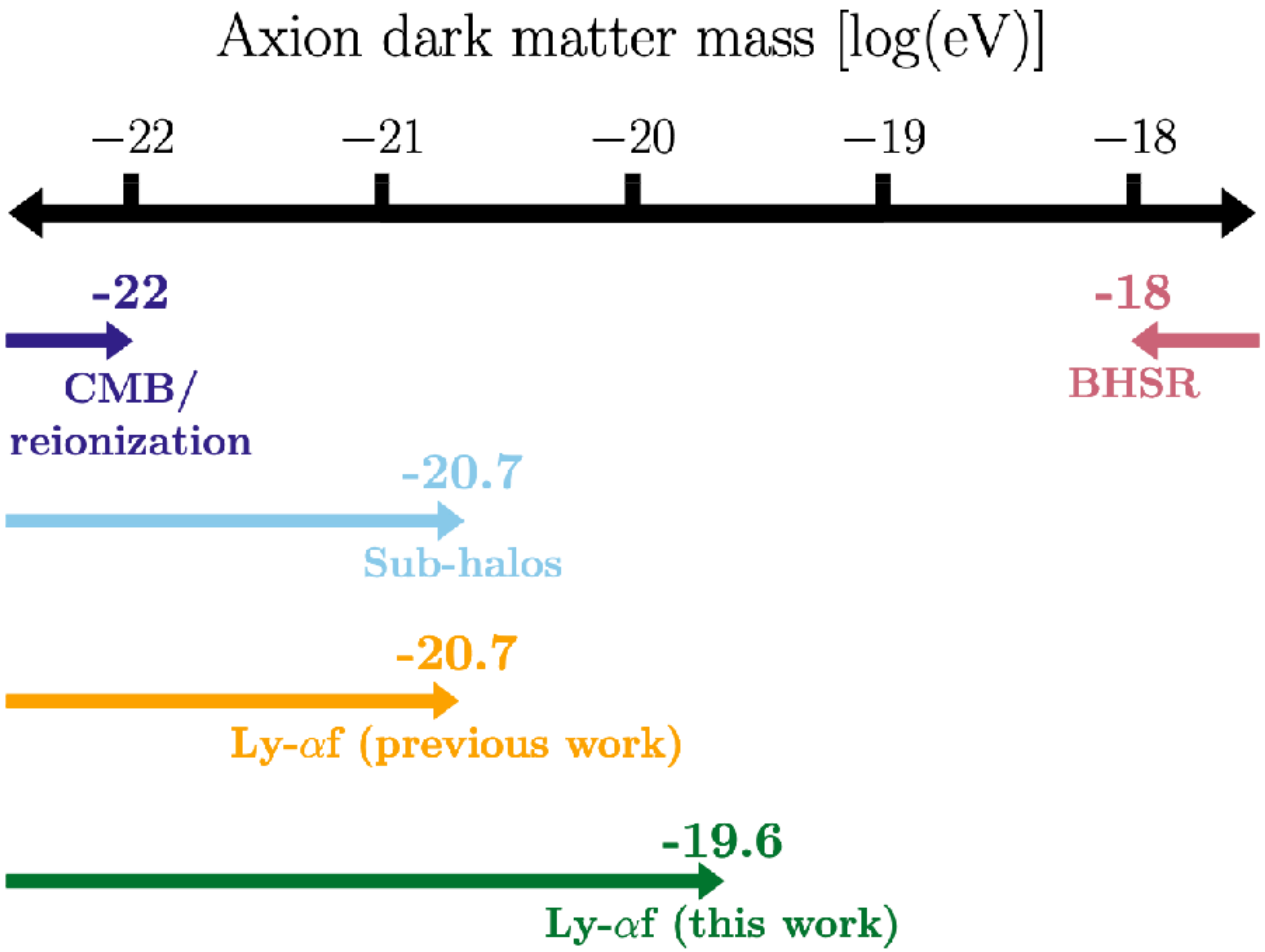

Snowmass 2021 White Paper Axion Dark Matter: arXiv:2203.14923
CROWS
ALPS-I
OSQAR
PVLAS
ALPS-II
ABRA 10 cm
SHAFT
CAST
Solar ν
Horizontal branch
White dwarfs
SN1987A (ν)
SN1987A (γ)
DSNALP
MWD X-rays
Neutron stars
Fermi-SNe
Mrk 421
Hydra
Star clusters
Fermi
MWD Pol.
M87
TOO RAD
LAMPOST
MUSE
VIMOS
Ionisation fraction
BBN+N_{eff}
Haloscopes
ADBC
DANCE
aLIGO
WISPLC
DM-Radio
SRF-m³
Black hole spins
IAXO
BREAD
BRASS
ORGAN
MADMAX
ALPHA
ADMX
FLASH
COBE/FIRAS
Leo T
EBL
X-rays
THESEUS
XMM-Newton
eROSITA
$|g_{a\gamma\gamma}|$ [GeV^{-1}]
m_a [eV]

FUZZY DARK MATTER

Linear theory predicts sharp cutoff in power spectrum due to quantum pressure

arXiv: 2007.13751

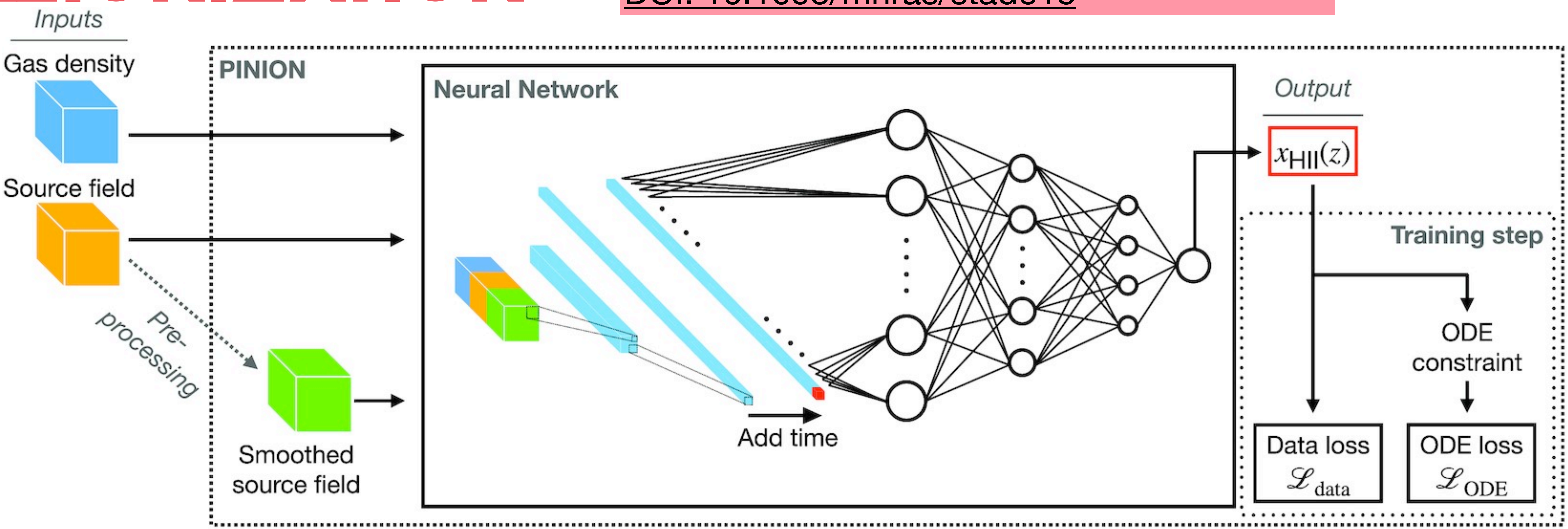




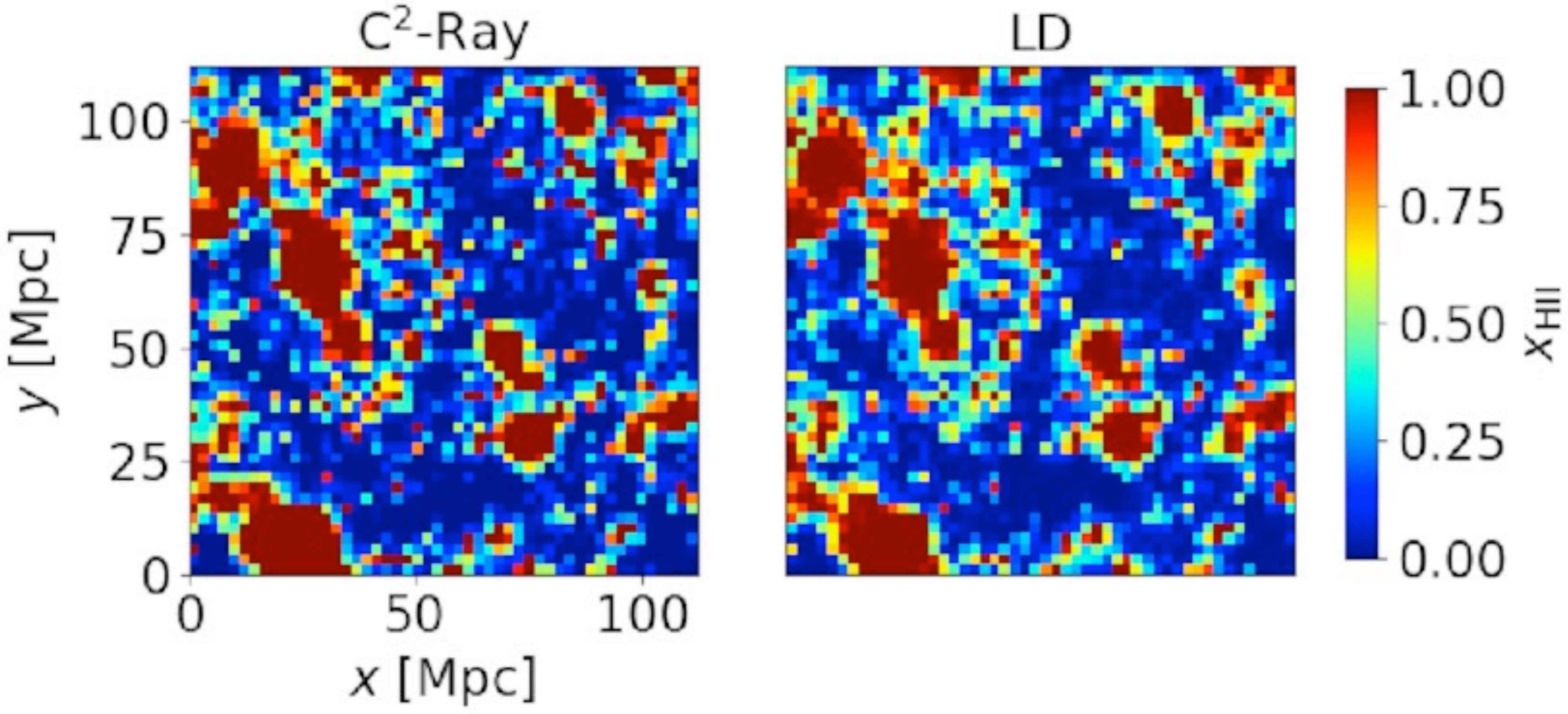
arXiv: 2007.12705

REIONIZATION

Korber, Bianco, Tolley, Kneib. MNRAS (2023)
DOI: 10.1093/mnras/stad615



$$\frac{dx_{HII}}{dt} = (1 - x_{HII})\Gamma - C\alpha_B n_H x_{HII}^2$$



DEEP LEARNING

Neural networks are universal function approximators

With enough neurons and an appropriate set of network weights and biases θ:

$$|y - \tilde{y}| < \epsilon$$

$$f(x) = y$$

$$\mathrm{NN}(x, \theta) = \tilde{y}$$

Model parameters θ minimized using loss function

$$\mathcal{L}_{\mathrm{MSE}} = \frac{1}{n} \sum_{i}^{n} (\tilde{y}_i - y_i)^2$$

LEARNING BIAS

Let's go back to the classic MSE loss

$$f(x) = y$$

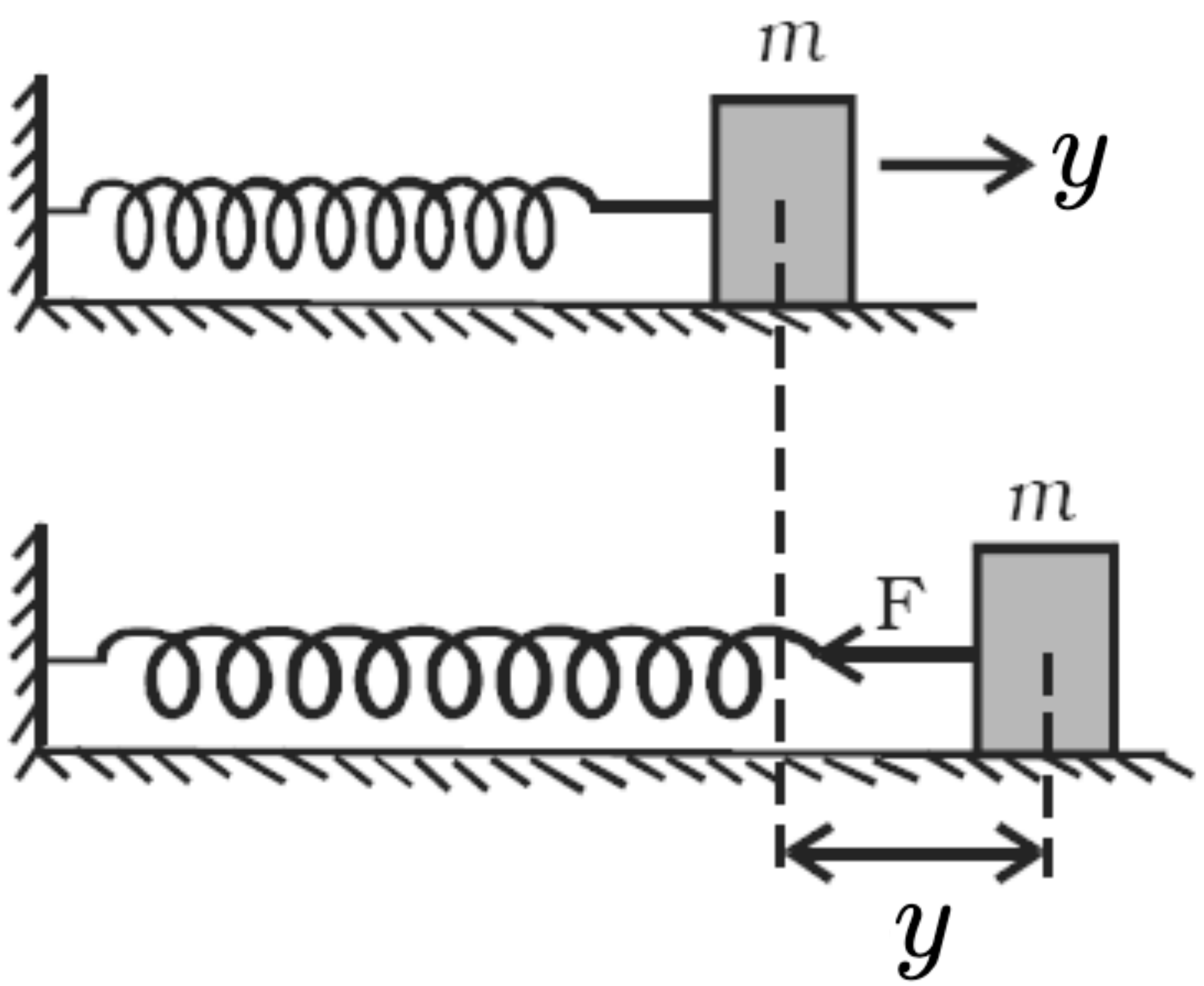
$$\mathrm{NN}(x, \theta) = \tilde{y}$$

$$\mathcal{L}_{\mathrm{MSE}} = \frac{1}{n}\sum_{i}^{n}(\tilde{y}_i - y_i)^2 = \frac{1}{n}\sum_{i}^{n}(\mathrm{NN}(x_i, \theta) - y_i)^2$$

Limited by sampling of x and y

LEARNING BIAS

A simple physics example



Harmonic oscillator if x is time and y is displacement

$$m\frac{\partial^2 y}{\partial x^2} + ky = 0$$

$f(x) = y$ obeys this dynamical equation, can use this to constrain the NN

LEARNING BIAS

A simple physics example

$$m\frac{\partial^2 y}{\partial x^2} + ky = 0$$

Does not need samples of y!
=> unsupervised learning

$$\mathcal{L}_{\mathrm{ODE}} = \frac{1}{n}\sum_{i}^{n}(m\frac{\partial^2 \tilde{y}_i}{\partial x^2} + k\tilde{y}_i)^2 = \frac{1}{n}\sum_{i}^{n}(m\frac{\partial^2}{\partial x^2}\ \mathrm{NN}(x_i,\theta) + k\ \mathrm{NN}(x_i,\theta))^2$$

If Loss=0 then network solves PDE (neural solver)

PHYSICS-INFORMED NEURAL NETWORK

https://www.brown.edu/research/projects/crunch/home

Use physics-based constraints from PDEs to make network training more efficient and generalizable